DOI: https://doi.org/10.61841/kkg5h717 Publication URL: https://jarm-s.com/MS/index.php/MS/article/view/58

NOVEL ROGUE-LIKE PARABOLIC-SOLITONS OF THE KADOMTSEV-PETVIASHVILI EQUATION

Jie-Fang Zhang[1]*, Mei-Zhen Jin[2]

1) Institute of Intelligent Media Technology, Communication University of Zhejiang, Hangzhou 310018, Zhejiang, China.
2) Network and Data Center, Communication University of Zhejiang, Hangzhou 310018, Zhejiang, China.

Corresponding Author:
zhangjief@cuz.edu.cn

To Cite this Article:
Zhang, J.- fang, & Jin, M.- zhen. (2024). Novel rogue-like parabolic-solitons of the Kadomtsev- Petviashvili equation. Journal of Advance Research in Mathematics And Statistics (ISSN 2208-2409), 11(1), 99-108. https://doi.org/10.61841/kkg5h717

ABSTRACT:

We proposes a new self-similar transformation of the KP equation for mapping into KdV equation and finds its novel rogue-like parabolic solitons with the 'short-lived' , which is similar to the rogue wave in NLS equation for first time.
The new solutions may be useful in the theory of rogue waves in a prototypical example of rogue wave in the (2+1)-dimensional nonlinear wave models. These studies could be helpful to deepen our understandings and enrich our knowledge about rogue waves.

KEYWORDS:

KP equation, KdV equation, Self-Similar Transformation, Parabolic Soliton, Rogue-Like Wave.

PACS NUMBERS:

02.30.Ik, 42.65.Sf, 05.45.-a, 47.27.Sd

1. INTRODUCTION

Rogue waves are a typical natural phenomenon that can occur in a variety of different physical environments [1]. The study on the rogue waves has become one of the hot and important subjects in natural science [2]. They are characterized by large amplitude, high steepness, no warning, short lives, etc. Theoretically, rational solutions of nonlinear Schrödinger (NLS) equation play a major role in the study of rogue waves for deep water. Rogue waves appear also in many other physical fields where the NLS type systems can be used, especially in fluid mechanics[3-5],nonlinear optical systems[6,7], plasmas[8,9], Bose-Einstein condensates[10,11], turbulence[12], microwaves[13], super-fluids[14], atmosphere[15], communications[16], capillary systems[17], financial systems[18], particulate matter[17], and magnetic materials [20]. In the past decades, the research on rogue waves has not only abundant theoretical results [21-36], but also abundant experimental verification [37-43].

We know that the mathematical description of water waves in shallow waters and coastal areas is usually based on solutions of the Korteweg–de Vries (KdV) equation or the Kadomtsev-Petviashvili (KP) equations [44,45], while the mathematical description of water waves in the open ocean and deep water are described by the NLS equation. However, the NLS equation has solutions in the form of rogue waves [46] and those phenomena have been observed in water tanks [42]. Therefore, it is of both theoretical and practical value to search for rogue wave solutions of the KP equation or the KdV equation as same those of the NLS equation, because the extreme water wave events frequently hit beaches and coastal areas and cause significant damage and loss of life, after all, constitute nearly 70% of the total number of extreme water wave events [47]. Of course, it needs to be mentioned here, in order to describe the shallow water rogue wave in shallow waters and coastal areas, Abdel-Gawad et al. [48] made the first attempts to find rogue wave solutions of the complex KdV equation derived by Levi [49]. Recently, Ankiewicz et al. [50] further investigated formation of rogue waves in shallow water from the modified KdV equation by using the complex Miura transformation. However, little has not been done on this subject for the KP equation, overall.

The paper is organized as follows. We first proposed the idea of self-similar transformation and searched for constructing method of self-similar wave in the frame of the KP equation. which is mapped to the KdV equation in Section 2, and secondly, we study the two-dimensional self-similar rogue parabolic-soliton on a inclined plane background for the KP equation (1) in Section 3. Finally, our conclusions are presented in Section 4.

2. SELF-SIMILAR TRANSFORMATION

We consider the following KP equation:

$$\left[u_t + 6uu_x + u_{xxx}\right]_x + 3\sigma^2 u_{yy} = 0, \quad \sigma^2 = \pm 1, \tag{1}$$

where subscripts denote differentiation, which is of considerable importance both in physics and mathematics. Eq.(1) arises in many physical applications including weakly two-dimensional long waves in shallow water, the sign of σ^2 depends upon the relevant magnitudes of gravity and surface tension[2], which is classified as the KPI equation when $\sigma = -1$ and the KPII equation when $\sigma = 1$. Our goal is to research for a mapping relation between Eq. (1) and the KdV equation

$$U_\tau + 6UU_\xi + U_{\xi\xi\xi} = 0. \tag{2}$$

To connect solutions of Eq. (1) with those of Eq.(2), we introduce a new self-similar transformation in the form

$$u(x,y,t) = \rho(t)U\left(\xi(x,y,t),\tau(t)\right) + \alpha(t)x. \tag{3}$$

where $U(\xi,\tau)$ satisfies KdV equation (2), $\xi = \xi(x,y,t), \tau = \tau(t)$ and $\rho = \rho(t),\ \alpha = \alpha(t)$ are four undetermined functions of the specified variables. It should be mentioned that $\xi = \xi(x,y,t)$, $\tau = \tau(t)$ are called the self-similar variables, $\rho(t)$ is called a amplitude amplification factor, $\alpha(t)x$ is called the excitation background, respectively. It should also be emphasized that this self-similar transformation is analogous to the self-similar analysis technique of NLS equation type equations, but the key point here is to add an additional term, of which important value can be seen later.

Substituting (3) into (1), we can get

$$\begin{aligned}&\rho\xi_x\tau_t U_{\xi\tau}+6\rho^2\xi_x^2(U_\xi^2+UU_{\xi\xi})+\rho\xi_x^4U_{\xi\xi\xi\xi}+6\rho^2\xi_{xx}UU_\xi+6\rho\xi_x^2\xi_{xx}U_{\xi\xi\xi}\\&+(\rho_t\xi_x+\rho\xi_{xt}+12\alpha\rho\xi_x+3\sigma^2\rho\xi_{yy}+6\alpha\rho\xi_{xx}x+\rho\xi_{xxxx})U_\xi\\&+\rho(\xi_x\xi_t+3\sigma^2\xi_y^2+6\alpha\xi_x^2x+3\xi_{xx}^2+4\xi_x\xi_{xxx})U_{\xi\xi}+\alpha_t+6\alpha^2=0,\end{aligned}\tag{4}$$

Requiring $U(\xi,\tau)$ to satisfy Eq. (2) and $u(x,y,t)$ to be a solution of Eq.(1)，we get the set of equations

$$\xi_{xx}=0,\xi_{xxx}=\xi_{xxxx}=0,\tag{5}$$

$$\alpha_t+6\alpha^2=0,\tag{6}$$

$$\xi_x\xi_t+3\sigma^2\xi_y^2+6\alpha\xi_x^2x=0,\tag{7}$$

$$\rho_t\xi_x+\rho\xi_{xt}+3\sigma^2\rho\xi_{yy}+12\alpha\rho\xi_x=0,\tag{8}$$

$$\tau_t=\rho\xi_x=\xi_x^3.\tag{9}$$

It can be inferred from Eq. (5) and Eq. (7)

$$\xi(x,t)=\kappa(t)x+\iota(t)y^2+\gamma(t)y+\omega(t),\tag{10}$$

where $\kappa(t),\iota(t),\gamma(t),\omega(t)$ are four undetermined functions of the specified variables t. Substituting Eq. (9) into Eqs.(5)–(8) and after some algebra yields

$$\begin{aligned}&\xi(x,y,t)=\frac{\kappa_0}{1+6\alpha_0t}x+\frac{\kappa_0\alpha_0}{\sigma^2(1+6\alpha_0t)^2}y^2+\frac{\gamma_0}{(1+6\alpha_0t)^2}y-\frac{3\sigma^2\gamma_0^2(1+3\alpha_0t)t}{\kappa_0(1+6\alpha_0t)^2}+\xi_0,\\&\tau=\frac{\kappa_0^3(1+3\alpha_0t)t}{(1+6\alpha_0t)^2}+\tau_0,\end{aligned}\tag{11}$$

$$\rho(t)=\frac{\kappa_0^2}{(1+6\alpha_0t)^2}.\tag{12}$$

where $\kappa_0,\alpha_0,\beta_0,\gamma_0$ and ξ_0,τ_0 are the free integration constants. Without loss of generality we can choose $\xi_0=\tau_0=0$.

According to the above, we get a SST (3) with expressions (11) and (12) for the KP equation, which may become into the KdV equation. Thus, the known solutions of the KdV equation can all be mapped to the solutions of the KP equation.

It is worth mentioning that the similarity reduction that the KP equation can been reduced to the KdV equationand the Boussinesq equation except for to the first and second Painlevé equation by means of infinitesimal transformation of Lie's method [51]. In particular, the similarity reductions of the KP equation are thoroughly studied by the direct reduction method [52-54]. But no above problems were yet involved.

3. ROGUE-LIKE PARABOLIC-SOLITONS

In the following, we use the SST to derive the self-similar parabolic-solitons(PS), which here are called as the rogue-like PS, of the KP equation from various solitons existed in KdV equation.

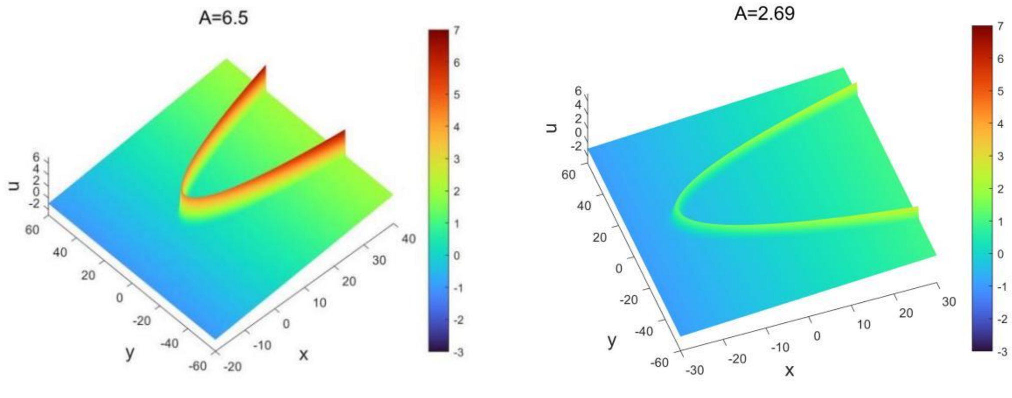
As the first application, we consider the single soliton solution of the KdV equation as follows[55]

$$U(\xi,\tau)=\frac{1}{2}k^2\,\mathrm{sech}^2\left[\frac{1}{2}\left(k\xi-\omega\tau\right)\right], \tag{13}$$

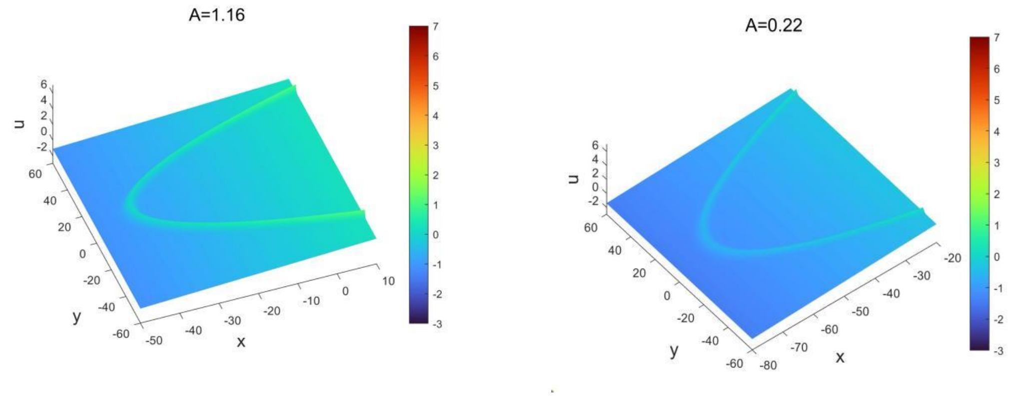
where k are an arbitrary real parameters, while $\omega=-k^3$. By virtue of the SST (3) with Eqs.(11)-(12) and the single soliton (13) of the KdV equation (2), we find the single self-similar parabolic solutions of the KP equation (1) as follows

$$\begin{aligned} u(x,y,t)=&\frac{k^2\kappa_0^2}{2(1+6\alpha_0\sigma^2 t)^2}\mathrm{sech}^2\left[\frac{k}{2(1+6\alpha_0 t)}\left(\kappa_0 x+\frac{\kappa_0\alpha_0}{\sigma^2(1+6\alpha_0 t)}y^2+\frac{\gamma_0}{1+6\alpha_0 t}y\right.\right.\\ &\left.\left.-\frac{3\sigma^2\gamma_0^2+\kappa_0^4 k^3}{\kappa_0}\frac{(1+3\alpha_0 t)t}{(1+6\alpha_0 t)}\right)\right]+\frac{\alpha_0}{1+6\alpha_0 t}x. \end{aligned} \tag{14}$$

Solution (14) involves six free parameters $\alpha_0,\gamma_0,\sigma,\kappa_0,k$. Fig.1 and Fig. 2 displays the solution (14) of and KPI equation ($\sigma^2=1$) and the KPII equation ($\sigma^2=1$) in the (x,y) -plane for fixing $\alpha_0=0.05,\beta_0=1,\gamma_0=1,$ $\kappa_0=3,k=1,\varphi_0=0$, respectively.



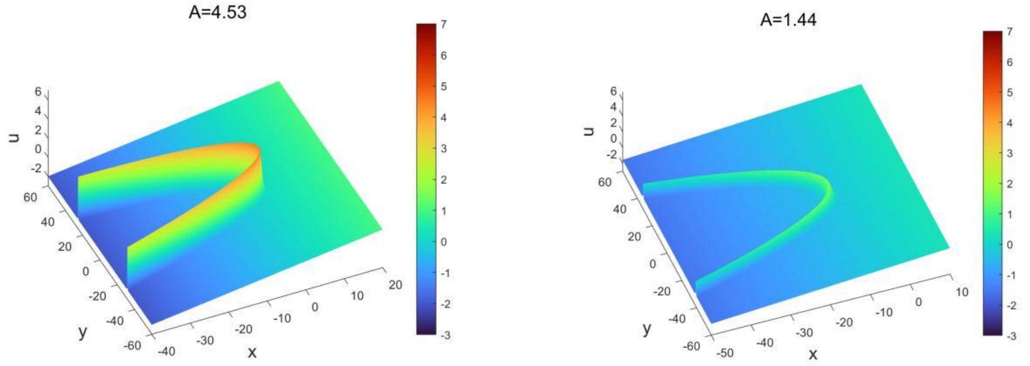
(a) (b)



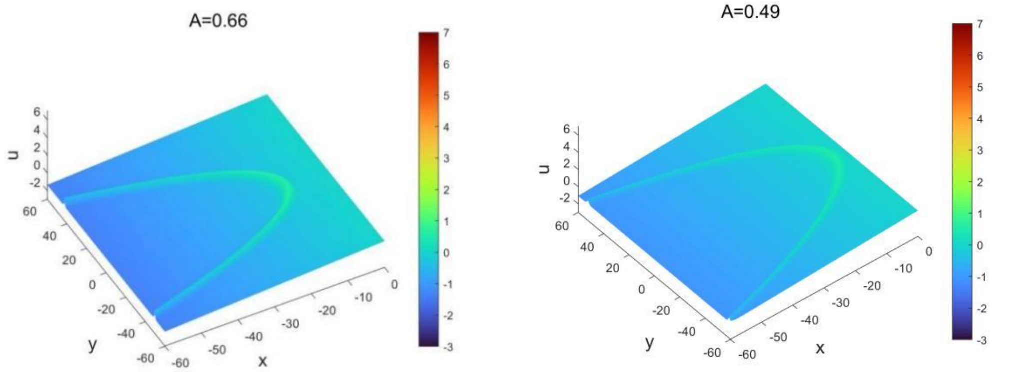
(c) (d)

Fig.1. (Color online.) Evolution of rogue-like parabolic soliton of KPI equation.

The parameters is chosen as $\sigma^2=-1,\alpha_0=0.05,\gamma_0=1\kappa_0=3,k=1,\varphi_0=0.$ (a) $t=0$, (b) $t=2$, (c) $t=4$, (d) $t=6$.



(a) (b)



(c) (d)

Fig. 2. (Color online.) Evolution of rogue-like single parabolic-soliton of KPII equation.

The parameters is chosen as $\sigma^2=1,\alpha_0=0.05,\gamma_0=1\kappa_0=3,k=1,\varphi_0=0.$ (a) $t=0$, (b) $t=2$, (c) $t=4$, (d) $t=6$.

It can be seen from Figure 1 and Figure 2 that the amplitudes of the single parabolic-soliton of the KP equation(1), which is mapped from the soliton of the KdV equation (2), can be arbitrarily large and decaying quickly with time. They show shows a strong ‘short-lived’ characteristics similar to the rogue wave in NLS equation. Here we can call them the moving rogue-like parabolic soliton of the KP equation in the (x, y) -plane.

As the second application, we consider N-soliton solutions of the KdV equation (2). By means of the classical Hirota bilinear method, the N line solitons of can be expressed as[56]

$$\tau) = 2\ln\left[\sum_{\mu=0,1}\exp\left(\sum_{i=1}^{N}\mu_i\Xi_i + \sum_{1\le i<j}^{N}\mu_i\mu_j a_{ij}\right)\right]_{\xi\xi}, \tag{15}$$

where

$$_{i}\xi - k_i^3\tau, \exp(a_{ij}) = A_{ij} = \left(\frac{k_i - k_j}{k_i + k_j}\right)^2. \tag{16}$$

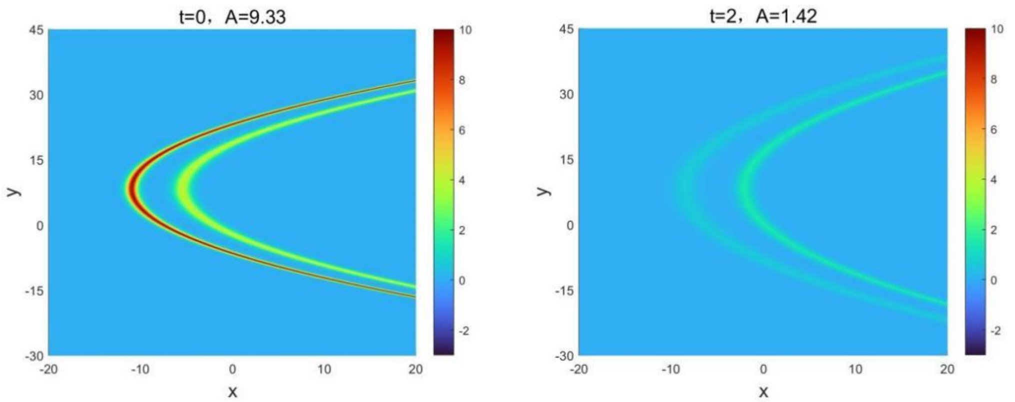
Similarly, under the SMT (3) with Eqs.(11) and Eq.(12), the self-similar N parabolic solitons of the KP equation (1) can be written as

$$\begin{aligned} u(x,y,t) &= \frac{\kappa_0^2}{(1+6\alpha_0 t)^2}U(\xi,\eta,\tau) + \frac{\alpha_0}{1+6\alpha_0 t}x \\ \frac{2\kappa_0^2}{+6\alpha_0 t)^2}&\ln\left[\sum_{\mu=0,1}\exp\left(\sum_{i=1}^{N}\mu_i\Xi_i + \sum_{1\le i<j}^{N}\mu_i\mu_j a_{ij}\right)\right]_{\xi\xi} + \frac{\alpha_0}{1+6\alpha_0 t}x, \end{aligned} \tag{17}$$

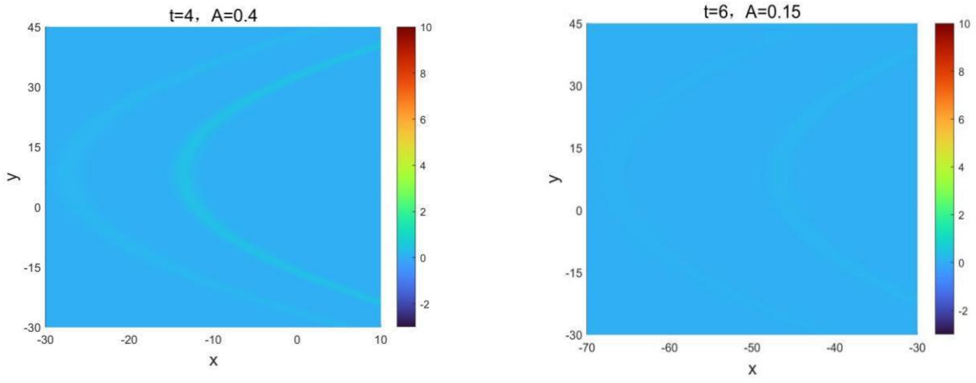
where

$$\begin{aligned} \Xi_i(x,y,t) &= k_i\xi - k_i^3\tau \\ &= \frac{k_i}{1+6\alpha_0 t}\left(\kappa_0 x + \frac{\kappa_0\alpha_0}{\sigma^2(1+6\alpha_0 t)}y^2 + \frac{\beta_0}{1+6\alpha_0 t}y - \frac{3\sigma^2\gamma_0^2 + k_i^2\kappa_0^4}{\kappa_0}\frac{(1+3\alpha_0 t)t}{(1+6\alpha_0 t)^2}\right) + \varphi_0. \end{aligned} \tag{18}$$

As another illustration, we discuss the double solitons of the KP equation (1) for solution (17) when $N = 2$. Because it involves six free parameters $\sigma, \alpha_0, \beta_0, \gamma_0$, $\kappa_0, k_1, k_2, \varphi_0$ to control the different types of self-similar parabolic-solitons propagation. Figure 3 and Figure 4 display the solution (17) of the KPI equation ($\sigma^2 = -1$) and the KPII equation ($\sigma^2 = 1$) in the (x, y)-plane for fixing $\alpha_0 = 0.05, \beta_0 = 1, \gamma_0 = 1$, $\kappa_0 = 3, k = 3, \varphi_0 = 0$, respectively.

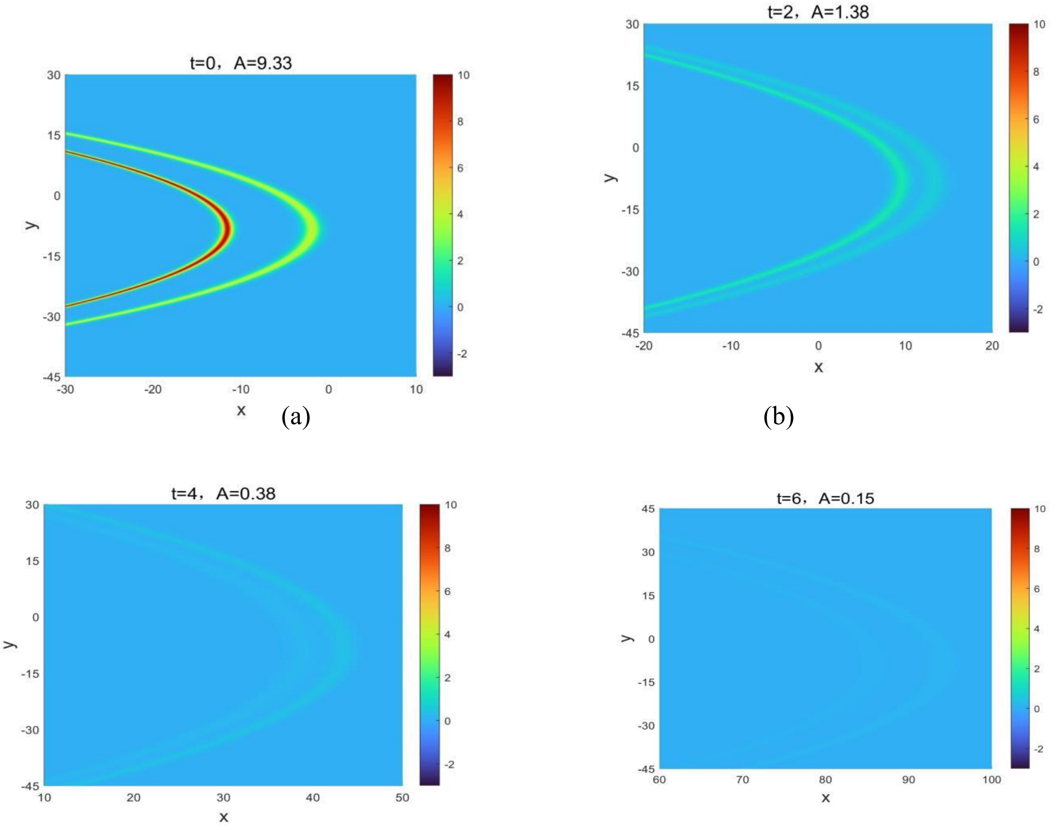


(a) (b)



(c) (d)

Fig. 3. (Color online.) Contour plot of rogue-like double parabolic-soliton of KPI equation. The parameters are chosen as $\sigma^2=-1, \alpha_0=0.05, \beta_0=1, \gamma_0=1, \kappa_0=1.2, k_1=3, k_2=2, \varphi_0=-1$. (a) $t=0$, (b) $t=2$, (c) $t=4$, (d) $t=6$.



(a) (b) (c) (d)

Fig.4. (Color online.) Contour plot of rogue-like double parabolic-soliton of KPII equation.

The parameters are chosen as $\sigma^2=1, \alpha_0=0.05, \beta_0=1, \gamma_0=1, \kappa_0=1.2, k_1=3, k_2=2, \varphi_0=-2$. (a) $t=0$, (b) $t=2$, (c) $t=4$, (d) $t=6$.

It can also be seen from Figure 3 and Figure 4 that the amplitudes of the double parabolic-solitons of the KP equation(1), which is mapped from the double soliton of the KdV equation (2), can be arbitrarily large and decaying quickly with time. A strong characteristics of the 'short-lives' similar to the rogue wave cluster in NLS equation is manifested. Here we can call them the moving rogue-like double parabolic solitons of the KP equation in the (x, y) -plane.

4. CONCLUSION

To summarize, we have established a novel SST and discovered a kinds of novel rogue-like parabolic solitons of the KP equation. Our results show also definitively that the amplitude of the wave controlled by several parameters can be large and decay very quickly in a short time, so that it just describes the characteristics of rogue waves.

The presented results could not directly be generalized to the other (2+1)-dimensional nonlinear evolution models, but it can bring some enlightenment for studying various integrable and nonintegrable nonlinear models by using of the self-similar transformation technology. The significance of our findings is not restricted to water rogue waves, which can be applied to nonlinear optics and to other fields where the KP equation is the governing equation. We surmise that new experiments characterized by KP equation will be easier to implement and the predictions of the present work could be verified.

REFERENCES

[1] Onorato, M., Residori, S., Bortolozzo, U., Montina, A., Arecchi, F., Rogue waves and their generating mechanisms in different physical contexts. Phys. Rep. **528**(2013)47–89.

[2]Ginzburg, N. S., Rozental, R. M., Sergeev, A. S., Fedotov, A. E., Zotova, I. V., Tarakanov V. P., Generation of Rogue Waves in Gyrotrons Operating in the Regime of Developed Turbulence. Phys. Rev. Lett. 119(2017)034801.

[3]Onorato M., Osborne, A. R., Serio, M., Bertone, S., Freak Waves in Random Oceanic Sea States. Phys. Rev. Lett. **86**(2001) 5831.

[4]Kharif, C., Pelinovsky, E., Slunyaev, A., Rogue Waves in the Ocean. Springer-Verlag, Berlin(2009).

[5]Adcock, T. A. A., Taylor, P. H.: The physics of anomalous ('rogue') ocean waves. Rep. Prog. Phys.**77**(2014)105901.

[6]Dudley, J. M., Genty, G., Eggleton, B. J., Harnessing and control of optical rogue waves in supercontinuum generation. Opt. Express 16(2008)3644–3651.

[7]Kasparian, J., Bejot , P., Wolf, J. P., Dudley, J. M., Optical rogue wave statistics in laser filamentation. Opt. Express **17** (2009)12070–12075.

[8]Shukla, P. K., Moslem, W. M., Alfvénic rogue waves. Phys. Lett. A **376**(12-13)(2012)1125 –1128.

[9]Tsai, Y. Y., Tsai, J. Y., Lin. I., Generation of acoustic rogue waves in dusty plasmas through three-dimensional particle focusing by distorted waveforms. Nature Phys. **12(6)**(2016)573-577

[10] Bludov, Y. V., Konotop, V. V., Akhmediev, N., Matter rogue waves. Phys. Rev. A **80**(2009) 033610.

[11] Vinayagam, P. S., Radha, R., Porsezian, K.,Taming rogue waves in vector Bose–Einstein condensates. Phys. Rev. E 88(2013)042906.

[12] Ginzburg, N. S., Rozental, R. M., Sergeev, A. S., Fedotov, A. E., Zotova, I. V., Tarakanov, V. P., Generation of rogue waves in gyrotrons operating in the regime of developed turbulence. Phys. Rev. Lett. **119**(2017) 034801.

[13] Hohmann, R., Kuhl, U., Stockmann, H. J., Kaplan, L., Heller, E. J., Freak waves in the linear regime: a microwave study. Phys. Rev. Lett. **104** (2010) 093901.

[14] Ganshin, A. N., Efimov, V. B., Kolmakov, G. V., Mezhov-Deglin, L. P., McClintock, P. V. E., Observation of an inverse energy cascade in developed acoustic turbulence in superfluid helium. Phys. Rev. Lett. **101**(2008)065303.

[15] Stenflo, L., Marklund, M., Rogue waves in the atmosphere. J. Plasma Phys. **76**(2010)293 –295.

[16] Hammani, K., Kibler, B., Finot, C., Morin, P., Millot, G., Peregrine soliton generation and breakup in standard telecommunications fiber. Opt. Lett. **36** (2011)112–114.

[17] Shats, M., Punzmann, H., Xia, H., Capillary Rogue Waves. Phys. Rev. Lett. **104** (2010) 104503.

[18] Yan, Z. Y., Vector financial rogue waves. Phys. Lett. **375(48)**(2011)4274-4279.

[19] Han, J. F., Liang, T., Duan, W. S., Possibility of the existence of the rogue wave and the super rogue wave in granular matter. Euro. Phys. J. E **42**(2019)5

[20] Copus, M. G., Camley, R. E., Creation of magnetic rogue waves. Phys. Rev. B, **102** (2020)220410(R).

[21] Soto-Crespo, J. M., Devine, N., Akhmediev, N., Integrable turbulence and rogue waves: breathers or solitons ?. Phys. Rev. Lett. **116**(2016)103901.

[22] Akhmediev, N., Dudley, J. M., Solli, D. R., Turitsyn, S. K., Recent progress in investigating optical rogue waves. J. Opt. **15**(2013) 060201.

[23] Ohta, Y., Yang, J. K., General high-order rogue waves and their dynamics in the nonlinear Schrödinger equation. Proc. R. Soc. Lond. Ser. A **468**(2012)1716–1740.

[24] Christian, K., Efim, P., Physical mechanisms of the rogue wave phenomenon. Euro. J. Mech. / B Fluids **22(6)**(2003) 603-634.

[25] Wang L H, He J S, Xu H, Wang J, Porsezian K., Generation of higher-order rogue waves from multibreathers by double degeneracy in an optical fiber, Phys. Rev. E, **95**(2017)042217.

[26] Dudley, J. M., Dias, F., Erkintalo, M., Genty, G., Instabilities, breathers and rogue waves in optics. Nature Photo. **8**(2014)755–764.

[27] Ankiewicz, A., Akhmediev, N., Rogue wave-type solutions of the mKdV equation and their relation to known NLSE rogue wave solutions. Nonlinear Dyn. **91**(2018) 1931–1938.

[28] Yin, H. M., Tian, B., Zhang, C.R., Du, X. X., Zhao, X. C., Optical breathers and rogue waves via the modulation instability for a higher-order generalized nonlinear Schrödinger equation in an optical fiber transmission system. Nonlinear Dyn. **97** (2019)843–852.

[29] Wang, Y. Y., Dai, C. Q., Zhou, G. Q., et al., Rogue wave and combined breather with repeatedly excited behaviors in the dispersion/diffraction decreasing medium. Nonlinear Dyn. **87**(2017)67–73.

[30] Temgoua, D. D. E., Tchoula Tchokonte, M. B., Maaza, M. et al., Contrast of optical activity and rogue wave propagation in chiral materials. Nonlinear Dyn. **95** (2019) 2691–2702.

[31] Mukam, S.P.T., Souleymanou, A., Kuetche, V.K., Bouetou, T.B., Generalized Darboux transformation and parameterdependent rogue wave solutions to a nonlinear shrodinger system. Nonlinear Dyn. **93(2)**(2018)373–383.

[32] Yu, W., Liu, W., Triki, H., Zhou, Q., Biswas, A., Belic, M.R., Control of dark and anti-dark solitons in the (2+1)-dimensional coupled nonlinear Schrodinger equations with perturbed dispersion and nonlinearity in a nonlinear optical system. Nonlinear Dyn. **97**(2019) 471–483.

[33] Dai, C. Q., Wang, Y. Y. & Zhang, J. F., Managements of scalar and vector rogue waves in a partially nonlocal nonlinear medium with linear and harmonic potentials. Nonlinear Dyn. **102**(2020)379–391.

[34] Ye, Y., Liu, J., Bu, L. et al., Rogue waves and modulation instability in an extended Manakov system. Nonlinear Dyn. **102** (2020)1801–1812.

[35] Mukam, S. P. T., Souleymanou, A., Kuetche, V. K. et al., Generalized Darboux transformation and parameter-dependent rogue wave solutions to a nonlinear Schrödinger system. Nonlinear Dyn. **93**(2018)373–383.

[36] Zhang, R. F. , Li, M. C., Yin, H.M., Rogue wave solutions and the bright and dark solitons of the (3+1)-dimensional Jimbo-Miwa equation. Nonlinear Dyn. **103**(2021)1071–1079.

[37] Birkholz, S., Nibbering, E. T. J., Spatiotemporal rogue events in optical multiple filamentation. Phys. Rev. Lett. **111**(2013)243903.

[38] Solli, D. R. Ropers, C., Koonath, P., Jalali, B., Optical rogue waves. Nature **450**(2007) 1054-1057.

[39] Solli, D. R., Ropers, C., Jalali, B., Active control of rogue waves for stimulated supercontinu.um generation. Phys. Rev. Lett. **101**(2008)233902

[40] Kibler, B., Fatome, J., Finot, C., Millot, G., Genty, G., Wetzel, B., Akhmediev, N., Dias, F., Dudley, J.M., Observation of Kuznetsov-Ma soliton dynamics in optical fiber. Sci. Rep. **2**(2012)463.

[41] Chabchoub, A., Hoffmannn N. P., Akhmediev, N., Rogue wave observation in a water wave tank. Phys. Rev. Lett.**106**(2011) 204502.

[42] Bailung, H., Sharma, S. K., Nakamura, Y., Observations of peregrine solitons in a multi- component plasma with negative ions. Phys. Rev. Lett. **107**(2011)255005.

[43] Xiong, H., Gan, J. H., Wu, Y., Kuznetsov-Ma Soliton dynamics based on the mechanical effect of light .Phys.

Rev. Lett. **119**(2017)153901.

[44] Kodama,Y., KP solitons in shallow water, J. Phys. A: Math. Theor. **43**(2010)434004.

[45] Korteweg, D. J., and De Vries, G., On the change of form of long waves advancing in a rectangular canal and on a new type of long stationary waves, Philos. Mag. **39**(1895)422.

[46] Shrira, V., and Geogjaev V., What makes the Peregrine soliton so special as a prototype of freak waves? J. Eng. Math. **67**(2010)11.

[47] Nikolkina, I., and Didenkulova,I., Rogue waves in 2006-2011, Nat. Hazards Earth Syst. Sci. **11** (2011)2913.

[48] Levi D., Levi-Civita theory for irrotational water waves in a one-dimensional channel and the complex Korteweg-de Vries equation,Teor. Mat. Fiz. **99**(1994)435. [Theor. Math. Phys. **99** (1994)705.]

[49] Abdel-Gawad,H. I.,Tantawy, M., and Elkhair. R. E. A., Waves Random Complex Media **26** (2016)397.

[50] Ankiewicz, A.,Bokaeeyan, M., and Akhmediev N., Shallow-water rogue waves: An approach based on complex solutions of the Korteweg–de Vries equation, Phys.Rev.E **99** (2019) 050201(R).

[51] Nishitani, T., and Tajiri, M., Similarity Solutions of the Kadomtsev-Petviashvili Equation, J. Phys. Soc. Jpn. **51** (1982)2350.

[52] Lou S.-Y., Similarity reductions of the KP equation by a direct method, J. Phys. A **24** (1991) 1455.

[53] Lou, S.-Y., Ruan H.-Y., Chen D.-F., and Chen W.-Z., Similarity solutions of the Kadomtsev- Petviashvili equation, J. Phys. A **23** (1990) L649.

[54] Clarkson, P. A., Nonclassical symmetry reductions for the Kadomtsev-Petviashvili equation, Physica D **49** (1991)257.

[55] Zabusky, N. J., and Kruskal, M. D., Shallow-water waves, the Korteweg-deVries equation and solitons, Phys. Rev. Lett. **15** (1965)240.

[56] Hirota, R., Exact solution of the Korteweg-de Vries equation for multiple collisions of solitons, Phys. Rev. Lett. **27**(1971)1192.