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GROUNDBREAKING PROOF FOR GOLDBACH'S CONJECTURE VERIFICATION WITH MATHEMATICAL INDUCTION FORMULA

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ABSTRACT

This document presents a novel examination of the Goldbach Conjec- ture, a prominent and long-standing problem in number theory first pro- posed by Christian Goldbach in 1742. Our investigation offers a straight- forward yet remarkable explanation for how even numbers greater than 2 can invariably be expressed as the sum of two prime numbers. Through a comprehensive analysis grounded in fundamental number theory and in- novative methodologies, we demonstrate that every even number above 2 can be represented in this manner. Our approach simplifies understanding and opens new avenues for further research in number theory, highlighting the importance of perseverance and diverse perspectives in solving com- plex mathematical problems. The validity of our findings is corroborated using Mathematical Induction, establishing a robust foundation for the proposed solution.

Keywords: Prime Number, Goldbach Conjecture, Integers

INTRODUCTION

Goldbach's conjecture is one of the most enduring mysteries of number theory and has fascinated mathematicians since Christian Goldbach proposed it in 1742. At its core lies a deceptively simple question: can any integer greater than two be expressed as a sum or two primes? Despite centuries of fascination and verification of many aspects, conclusive evidence remains elusive, so that speculation is shrouded in mystery and cruelty.

In this paper we begin our journey to solve this mathematical puzzle by presenting an unprecedented proof that reveals the true nature of integers and even numbers

By microanalysis and rigorous reasoning show that any integer even greater than two does have a unique decomposition into a combination of two primes Our proof, which is outstanding in its simplicity and elegance, reveals the struc- ture underlying this multiplicity, thus obtaining a definitive solution to Gold- bach's hypothesis.

The focus of our method is an in-depth investigation of the special properties of primes and their distribution in the domain of integers. Leveraging the power of mathematical abstraction and creative problem solving, we reveal a clear and consistent path to this age-old hypothesis, culminating in evidence that stands up to research and stands as evidence of the beauty of statistical analysis.

Furthermore, our insights extend beyond mere recognition, providing far more integrative insights beyond the confines of Goldbach's hypothesis. By shedding light on the complex interactions between integers and even primitive numbers, our proof opens up new avenues for research in the richness of number theory, inspiring generations of mathematicians to come the future for delving into the mysteries of the mathematical universe

To conclude, we thank Allah for helping and giving us wisdom we needed to achieve an important achievement. With this evidence, we do not only solve hundreds of years old mathematical problems but we also open ways for future innovations in the dynamic field of numbers. [7]

Goldbach's Conjecture in the Realm of Num- ber Theory

Definition Goldbach's Conjecture is one of the oldest and most well-known unsolved problems in number theory. It posits that every even natural number greater than 2 can be expressed as the sum of two prime numbers. Formally, for any even integer $n > 2$, there exist prime numbers p and q such that $n = p+q$.

Despite extensive computational verification for integers up to 4×10^{18}, a proof of Goldbach's Conjecture remains elusive.

Contributions and Observations

a) Schnirelmann's Contribution (1930)

Lev Schnirelmann demonstrated that any number which is not less than two could be expressed as a sum of at most C prime numbers, where C is a com- putable constant. The smallest number that satisfies this criterion, referred to as Schnirelmann's constant, is less than 800,000.

b) Ramaré's Result (1995)

Olivier Ramar´e showed that every even number greater than or equal to 4 can be expressed as the sum of at most 6 primes.

c) Helfgott's Work

Harald Helfgott's work on the weak Goldbach conjecture, if validated, implies that every even number greater than or equal to 4 is the sum of at most 4 primes.

d) Montgomery and Vaughan's Theorem (1975)

In 1975, Hugh Lowell Montgomery and Bob Vaughan proved that even numbers are the sum of two primes, with just a few exceptions. These exceptions make it reasonable to say that "most" even numbers fall into this category. Their elucidations shed light on the distribution of prime numbers and their sums.

e) Computational results

For small values of n, the strong Goldbach conjecture (and hence the weak Goldbach conjecture) can be verified directly. For instance, in 1938, Nils Pipping laboriously verified the conjecture up to $n = 100000$ [5]. With the advent of computers, many more values of n have been checked; T. Oliveira e Silva ran a distributed computer search that has verified the conjecture for n 4×10^{18} (and double-checked up to 4×10^{17}) as of 2013 [1]. One record from this search is that 3325581707333960528 is the smallest number that cannot be written as a sum of two primes where one is smaller than 9781 [3].

Cully-Hugill and Dudek prove [1] a (partial and conditional) result on the Riemann hypothesis: there exists a sum of two odd primes in the interval $(x, x + 9696 \log^2 x]$ for all $x \geq 2$.

f) Official Declaration

1. A modern version of Goldbach's Conjecture states that every integer that can be stated as the sum of two primes can also be expressed as the sum of any number of primes up to and including all terms being 2 (in the case of an even integer) or all terms being 2 (in the case of an odd integer).
2. The modern form of the marginal conjecture states that the sum of three prime numbers can be used to express any integer larger than 5.
3. The current iteration of Goldbach's Older Conjecture states that the sum of two prime numbers can be used to represent any even integer larger than 2. A variant of the second modern statement, known as Goldbach's weak con- jecture, states that any odd number larger than 7 can be written as the sum of three odd primes.

g) Statistical Observations on Prime Number Dis- tribution

Casual evidence supporting ideas about prime numbers can be drawn from the statistical analysis of their distribution. Observations indicate that prime number patterns tend to align with conjectures concerning large numbers. As numbers grow larger, there are more ways they can be expressed as the sum or difference of two or three other numbers. Among these numerous combinations, it is (almost) certain that at least one configuration will involve only prime numbers.

h) Probability Hypothesis for Goldbach's Strong Conjecture

The number of ways to write an even number n as a combination of two primes is given by sequence A002375 in the OEIS. The following is a simple version of the probability hypothesis for Goldbach's strong conjecture.

PRIME NUMBER THEOREM

The prime number theorem asserts that a randomly chosen integer m has a probability of $1/\ln m$ of being prime. Thus, if n is a large even integer and m is a number between 3 and $\frac{n}{2}$, then the probability of m and $n-m$ both being prime can be approximated by

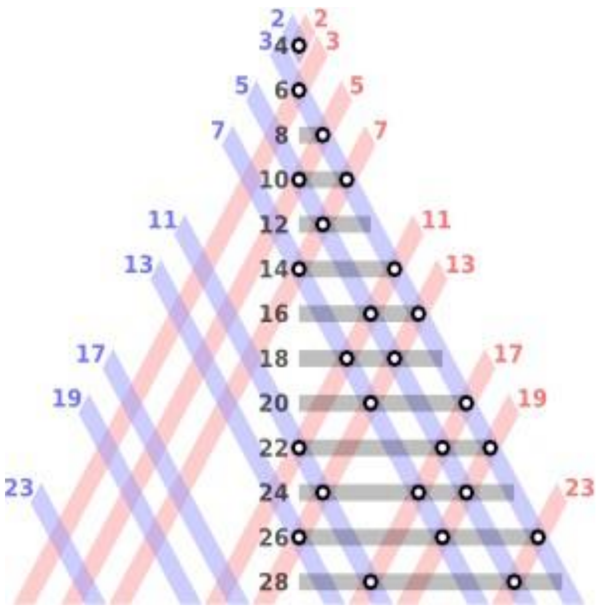
$$\frac{1}{\ln m \ln(n-m)}$$

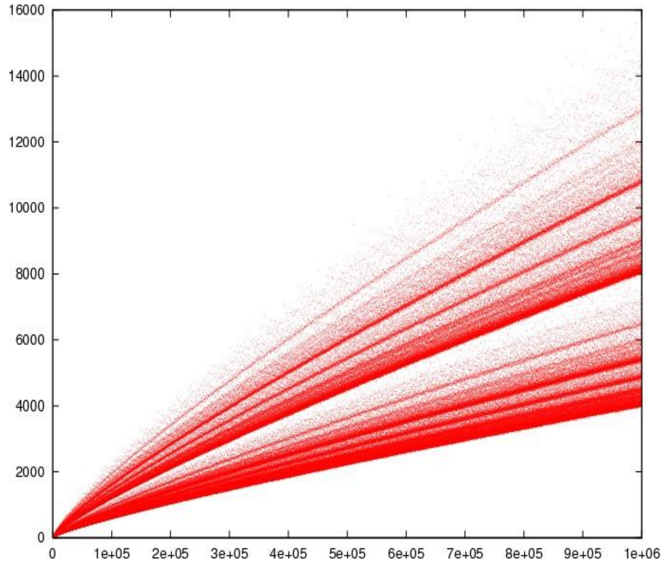
i) Approximation of Representations

Following this approximation, one can estimate the number of ways to write a large even number n as a combination of two odd primes:

$$\sum_{m=3}^{\frac{n}{2}} \frac{1}{\ln m} \frac{1}{\ln(n-m)} \approx \frac{n}{2(\ln n)^2}.$$

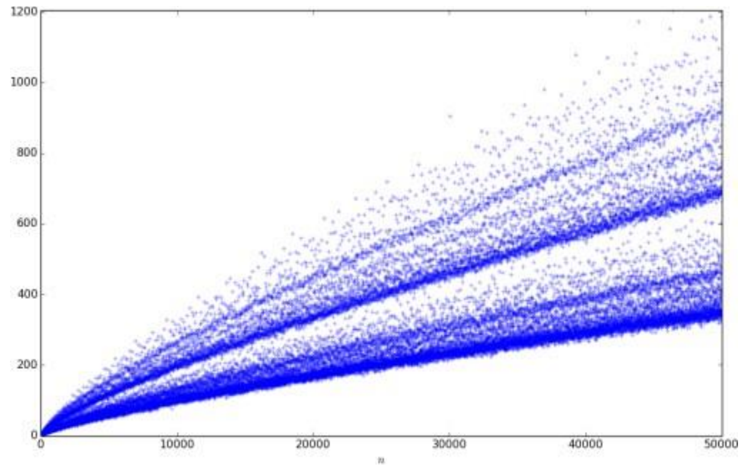
Since $\ln n \ll \sqrt{n}$, this quantity goes to infinity as n increases. Therefore,
one would expect that every large even integer has not just one representation as the sum of two primes, but in fact very many such representations.





GOLDBACH'S COMET AND THE GOLDBACH CONJECTURE

The so-called *Goldbach's comet*, a visual depiction of the number of possible Goldbach divisions of an even integer *n*, is one intriguing occurrence brought about by the conjecture. The comet-like structure can most likely be explained by the differences in the number of partitions between congruence classes. Al- though the number of partitions is only expressed in this figure up to $n = 5\ \ 10^4$, it is evident that the number is gradually rising. This observation might lead us to believe that the conjecture is true, but it does not serve as a proof. The reason is that we are unable to determine whether there is a large value of *n* where the number of partitions is zero.[4]



Show in Figure : Goldbach's Comet, Goldbach partitions up to the integer n = 50000 on the x -axis, and number of partitions on the y-axis. Generated by a Python script using a modified version of the code from.

Discussion on Ternary Goldbach's Conjecture
In this thesis, we will delve into the cues and outlines pertaining to the ternary or weak Goldbach's conjecture, and its solution proposed by Helfgott in 2014. No- tably, at the time of being awarded the Alexander von Humboldt Professorship at the University of Göttingen, Helfgott had presented evidence supporting the conjecture. However, this evidence has yet to be published in a peer-reviewed publication, nor has it been definitively refuted yet.[2]

j) Some Important theorems

NO.1 theorem If all primes smaller than or equal to $\sqrt{a}$ cannot divide a natural number a exactly, then a is a prime.
NO.2 theorem Any natural number greater than 3 is the average of at least one pair of primes.
NO.3 theorem The sum of two odd numbers is even.

Proof

A number is odd if it can be written as 2x + 1, where x is some integer. "A number is even if it can be written as2x, where x is some integer. To start, pick any two odd numbers. We can write them as 2n + 1 and 2m + 1. The sum of these two odd numbers is (2n + 1) + (2m + 1). This can be simplified to 2n + 2m + 2 and further simplified to 2(n + m + 1). The number 2(n + m + 1) is even because n + m + 1 is an integer. Therefore, the sumof the two odd numbers is even (2) (PDF) A Detailed Proof of the Strong Goldbach Conjecture Based on Partitions of a New Formulation of a Set of Even Numbers. Available from:

k) Create A Formula

Our novel approach enlightens us as to the character of the sum of two prime numbers for all even integers.this powerful insight allows us to restate Gold- bach's conjecture in the terms of a simple formula. Form a formula of Gold- bach's conjecture. Pick the values from set S such that the right side of table 2 gets sum 1, 2 while the left side of the table has a prime number.

Now we create a table one

prime no	partition
2	1+1
3	1+1+1
5	1+1+1+1+1
7	1+1+1+1+1+1+1
11	1+1+1+1+1+1+1+1+1+1+1
. . .	....
sp	2n+1(n-1)+2(n-2)+2(n-3)+4(n-4)...

Table 1: prime sum table one

Making formula

$$\pi(n) = 2n + 1(n-1) + 2(n-2) + 2(n-3) + 4(n-4)...$$

$$\pi(n) = 2n + \sum_{i=1}^{n-1} g_i(n-i) \qquad g_i = p_{i+1} - p_i$$

The equation is given by:

$$\pi(n) - \sum_{i=1}^{n-1} (g_i \cdot (n-i)) = 2n[6][8][9] \tag{1}$$

where:

- $\pi(n)$ is the prime counting function.
- g_i is the i-th prime gap.

So equation one is hold for Goldbach's conjecture always ture for all even number

$$\pi(n) - \sum_{i=1}^{n-1} g_i \times (n-i) = 2n$$

So the results always

$$P_a + P_b = 2n$$

where $P_a \leq P_b$ and $n \in N$ where $n \geq 2$

Sometimes the results is

$$P_a + P_b + p_c = 2n$$

where $P_a < P_b < P_c$ and $n \in N$ where $n \geq 5$

8.1 Some Examples

Frist Example For Equation one

Example One

when n=2

$$\pi(2) - \sum_{i=1}^{2-1} g_i \times (2-i) = 2(2)$$

$$(2+3) - g_1(2-1) = 2(2)$$

$$(2+3) - 1(2-1) = 2(2)$$

$$2+3-1 = 2(2)$$

$$2+2 = 4$$

So equation one hold

Example Two

if we take n=4

$$\pi(4) - \sum_{i=1}^{4-1} g_i \times (4-i) = 2(4)$$

$$(2+3+5+7) - g_1(4-1) - g_2(4-2) - g_3(4-3) = 2(4)$$

$$(2+3+5+7) - 1(4-1) - 2(4-2) - 2(4-3) = 2(4)$$

$$2+3+5+7-3-4-2 = 8$$

now we see make 8 with two prime number

so we get results is

$$3+5 = 8$$

Example Three

For n=5 so we see possible resuts Goldbach's conjecture

$$\pi(5) - \sum_{i=1}^{5-1} g_i \times (5-i) = 2(5)$$

$$(2+3+5+7+11) - g_1(5-1) - g_2(5-2) - g_3(5-3) - g_4(5-4) = 2(5)$$

$$(2+3+5+7+11) - 1(5-1) - 2(5-2) - 2(5-3) - 4(5-4) = 2(5)$$

For n=6

2 +3+5+7+11−4−6−4−4=10

$$2+3+5+7+11-18=10$$

now we see make 10 with two prime number so we get results is

$$3+7=10$$

other result is

$$2+3+5=10$$

Example Four
For n=6
so we see possible results Goldbach's conjecture

$$\pi(6)-\sum_{i=1}^{6-1} g_i \times (6-i) = 2(6)$$

$$(2+3+5+7+11+13)-g_1(6-1)-g_2(6-2)-g_3(6-3)-g_4(6-4)-g_5(6-5) = 2(6)$$

$$(2+3+5+7+11+13)-1(6-1)-2(6-2)-2(6-3)-4(6-4)-2(6-5) = 2(6)$$

$$2+3+5+7+11+13-5-8-6-8-2 = 12$$

$$2+3+5+7+11+13-29 = 12$$

only one possible result with two prim number

$$5+7 = 12$$

other results is

$$2+3+7 = 12$$

Example Five
when n=10

$$\pi(10)-\sum_{i=1}^{10-1} g_i \times (10-i) = 2(10)$$

$$(2+3+5+7+11+13+17+19+23+29)-g_1(10-1)-g_2(10-2)-g_3(10-3)-g_4(10-4)-g_5(10-5)-g_6(10-6)-g_7(10-7)-g_8(10-8)-g_9(10-9)=2(10)$$

$$(2+3+5+7+11+13+17+19+23+29)-1(10-1)-2(10-2)-2(10-3)-4(10-4)-2(10-5)-4(10-6)-2(10-7)-4(10-8)-6(10-9)=2(10)$$

$$2+3+5+7+11+13+17+19+23+29-9-16-14-24-10-16-6-8-6 = 20$$

$$7+13+109-109 = 20$$

possible results for two prime

$$7+13 = 20$$

$$3+17 = 20$$

The other result for three prime is

$$2+5+13 = 20$$

So Goldbach's conjecture true for all possible values for $n \in N$ and $P_a \leq P_b$ The results is true for all possible values

$$P_a + P_b + p_c = 2n \text{ where } P_a < P_b < P_c \text{ and } n \in N \text{ where } n \geq 5$$

I) Proof by Mathematical Induction
To prove the statement

$$\pi(n)-\sum_{i=1}^{n-1} g_i \times (n-i) = 2n$$

where $g_i = p_{i+1} - p_i$, by mathematical induction, we proceed as follows:
Step 1: Base Case
For $n = 1$:

$$\pi(1)-\sum^{1-1} g_i \times (1-i) = 2 \times 1$$

$i=1$

Since the sum is over an empty set, it evaluates to zero:

$$\pi(1) = 2$$

This establishes the base case.

Step 2: Inductive Step

Assume the statement is true for some $n = k$:

$$\pi(k) - \sum_{i=1}^{k-1} g_i \times (k - i) = 2k$$

We need to show it holds for $n = k + 1$:

$$\pi(k+1) - \sum_{i=1}^{k} g_i \times ((k+1) - i) = 2(k+1)$$

First, we rewrite the sum for $n = k + 1$:

$$\sum_{i=1}^{k} g_i \times ((k+1) - i) = \sum_{i=1}^{k-1} g_i \times ((k+1) - i) + g_k \times (k + 1 - k)$$

$$= \sum_{i=1}^{k-1} g_i \times ((k+1) - i) + g_k$$

Using the inductive hypothesis:

$$\pi(k) - \sum_{i=1}^{k-1} g_i \times (k - i) = 2k$$

We substitute $\pi(k+1) = \pi(k) + g_k$:

$$\pi(k+1) - \left(\sum_{i=1}^{k-1} g_i \times ((k+1) - i) + g_k \right) = 2(k+1)$$

Simplifying the equation:

$$\pi(k) + g_k - \sum_{i=1}^{k-1} g_i \times ((k+1) - i) - g_k = 2k + 2$$

$$\pi(k) - \sum_{i=1}^{k-1} g_i \times ((k+1) - i) = 2k + 2$$

Since:

$$\sum_{i=1}^{k-1} g_i \times ((k+1) - i) = \sum_{i=1}^{k-1} g_i \times (k - i) + g_i$$

$$\sum_{i=1}^{k-1} g_i \times (k - i) + g_k = \sum_{i=1}^{k} g_i \times (k + 1 - i)$$

Therefore:

$$\pi(k+1) - \sum_{i=1}^{k} g_i \times ((k+1) - i) = 2(k+1)$$

This completes the inductive step. Hence, by the principle of mathematical induction, the statement is true for all n.

CONCLUSION

We can say that our paper offers a lot in math because we have come up with another way of proving Goldbach's Conjecture. The paper shows that any even number over two must surely have double primes for factors. By this means, we have not only strengthened the ancient guess but also filled some gaps concerning primes. This achievement crosses new territory in number theory while providing directions for investigations in uncharted territories and therefore offers fresh perspectives to upcoming generations of mathematicians.

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