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GENERALIZED SOFT INTERSECTIONAL IDEALS IN TERNARY SEMIRINGS

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Abstract: -
In this paper we introduce the notions of soft intersectional ternary subsemirings and soft intersectional ideals in ternary semirings. We also discuss some basic results associated with these notions. In the last part of the paper, we characterize regular and weakly regular ternary semirings by their soft intersectional ideals.

Key words: *Ternary semirings, regular ternary semirings, weakly regular ternary semirings, soft intersectional ideals in ternary semirings.*

I.

1. INTRODUCTION

Molodtsov,[24] in 1999, came forward with a novel mathematical tool (named, soft sets) to analyze the uncertain situations, where complete information are not available and accentuated the need of usage of soft sets in various fields with interesting examples. In the second phase, after Molodtsov, the names of P. K. Maji, M. I. Ali, M.
Shabir, F. Feng, Y. B. Jun, H. Aktas, N. Cagman, A. Sezgin and A. O. Atagun are of vital importance in the development of soft logic to make it able enough to be used in applied fields, see [3, 5, 6, 8, 9, 12, 13, 14, 15, 16, 22, 23, 25, 26, 29].
The idea of semirings have been focal point of study by mathematicians and has set its worth in information sciences. The role of ideals in semirings is of cardinal importance and serve greatly in numerous objects. H. S. Vandiver[28] primarily generated the structure of semirings. J. Ahsan's[2] introduction and characterization of weakly regular semirings was coined in terms of their ideals.
D. H. Lehmer[17] was first introduced ternary algebraic structure in 1932, called triplexes. Next, in 1971, Lister[18] characterized the additive semigroups of rings, which are closed under the triple ring product as ternary rings. Later, Duta and Kar, and Bhambri[10, 11, 7] theorized the notion of ternary semirings and weakly regular ternary semirings, respectively.

F. Feng and Y. B. Jun[12] ware the pioneers to define the soft semirings and soft ideals in soft semirings, in 2008. Later, in 2014, X. Ma and J. Zhan[19, 20] contributed by their work on Hemirings through soft intersection h-ideals and h-hemiregular hemirings via (M,N)-SI-h-ideals. A good amount of work already been done on soft semigroups and soft rings[1, 4] in 2010, by U. Acar and M. I. Ali. Next, S. Z. Song and H. S. Kim[27] worked on soft intersection semigroups. After that, T.
Mahmood and U. Tariq [21] accelerated the work in the same sense by introducing soft intersection k-semirings and detailed work in this regard.

In this paper, we are going to introduce the concept of soft intersection ternary semirings. Next, we shall characterize the regular ternary semirings by means of soft intersection ternary ideals, and weakly regular ternary semirings in the same terminology.

2. Preliminaries

2.1 Definition

A set $S \neq \Phi$ with a binary operation addition "+" and a ternary multiplication "." , denoted by juxtaposition, is said to be a ternary semiring S , if it satisfies the following conditions:

$(i)\ (mno)pq = m(nop)q = mn(opq)$

$(ii)\ (m+n)pq = mpq + npq$

$(iii)\ m(n+p)q = mnq + mpq$

$(iv)\ mn(p+q) = mnp + mnq,$

for all $r, s, t, u, v \in S$. It shall be denoted by Ŧ$_S$.
We can see that any semiring can be reduced to Ŧ$_S$.
However, the converse may not hold in general.

2.2 Example

The set of –ve integers is Ŧ$_S$ under usual "+" and ternary "." . Note that it is not semi-ring under usual
"+" and ".".

2.3 Definition

An additive subsemigroup J of S is called a ternary left (right, lateral) ideal of S if uvj $(juv, ujv) \in J$,
$\forall u, v \in S$ and $j \in J$. It shall be denoted by Ŧ$_L i$ (Ŧ$_R i$, Ŧ$_E i$).
If J is both Ŧ$_L i$ and Ŧ$_R i$ of S, then J is called two-sided ideal of S. It shall be denoted by Ŧ$_T i$.
If J is an Ŧ$_T i$ and Ŧ$_E i$ of S , then J is called an ideal of S. It shall be denoted by Ŧ$_I$.

2.4 Definition

Ŧ$_S$ with 1 means $11u = 1u1 = u11 = u$, $\forall\ u \in S$.

2.5 Definition

Let $(R,+,.)$ be a Ŧ$_S$. An element $l \in R$ is called multiplicatively idempotent if $l^3 = l$.

2.6 Definition

S is called regular if for any $r \in S$, there exist
$u \in S$ such that $r = rur$ or $r \in rSr$, $\forall r \in S$. It shall be denoted by Ŧ$_{vNr\text{-}}$.

2.7 Theorem

S is regular if and only if $A \cap B \cap C = AB$ for A,
B and C as Ŧ$_R i$, Ŧ$_L i$ and Ŧ$_E i$ of S.

2.8 Definition

An element $a \in S$ is called right weakly (left weakly, lateral weakly) regular if $a \in (aS)^3 S$ ($a \in S(aS)^3$, $a \in SSaSaSS$), respectively.
A ternary semiring S is called right weakly (left weakly, lateral weakly) regular if all its elements are right weakly (left weakly, lateral weakly) regular, respectively. It shall be denoted by Ŧ$_R w$ r- (Ŧ$_L w$ r- ,
Ŧ$E w$ r-).
From now to onward, if otherwise stated, S will always denote a Ŧ$_S$. Further, for undefined terms and notions of S , see [17].
If U is initial universe, E is a set of parameters and A, B, C, ... are subsets of E. Then we have:

2.9 Definition [24]

A soft set (φ, A) over U means that φ is a mapping $\varphi: A \to P(U)$. Then we will write here (φ, A, U) instead of writing "(φ, A) is soft set of A over U ", if otherwise stated.

2.10 Definition [27]

Let (φ, A, U) and $\rho \in P(U)$. Then the set $i_A(\varphi;\rho) = \{w \in A : \rho \subseteq \varphi(w)\}$ is called ρ -inclusive set of (φ, A).

2.11 Definition

For $\Phi \neq A \subseteq S$, the characteristic soft set of S over U is $C_A(w) = \begin{cases} \Phi & if \ w \notin A, \\ U & if \ w \in A. \end{cases}$ C_S is known as identity soft set of S over U.

3. Main Results

Here we take $E=S$, if otherwise stated.

3.1 Definition

For (φ_1, S, U) and (φ_2, S, U), the sum $(\varphi_1 + \varphi_2, S, U)$ is defined by

$$(\varphi_1 + \varphi_2)(w) = \bigcup_{w=a+b} \{\varphi_1(a) \cap \varphi_2(b)\}, \ \forall \ w \in S.$$

3.2 Definition

For (φ_1, S, U), (φ_2, S, U) and (φ_3, S, U), the product $(\varphi_1\varphi_2\varphi_3, S, U)$ is defined by

$$(\varphi_1\varphi_2\varphi_3)(w) = \bigcup_{w=a_ib_ic_i} [\bigcap_i \{\varphi_1(a_i) \cap \varphi_2(b_i) \cap \varphi_3(c_i)\}],$$

$\forall \ w \in S.$

In this chapter, if otherwise stated, we will use the consideration $\Phi \subseteq M \subset N \subseteq U.$.

3.3 Definition

(φ, S, U) is called $(M N,)$ soft intersectional subsemiring of S if

(i) $\varphi(r+s) \cup M \supseteq \varphi(r) \cap \varphi(s) \cap N,$

(ii) $\varphi(rst) \cup M \supseteq \varphi(r) \cap \varphi(s) \cap \varphi(t) \cap N,$

$\forall \ r, \ s, \ t \in S.$ It will be denoted by $\varphi^t_{[M,N]SI\text{-}S}$.

3.4 Definition

(φ, S, U) is called $(M\,N,)$ soft intersectional left (right, lateral) ideal of S if

(i) $\varphi(r+s)\cup M \supseteq \varphi(r)\cap\varphi(s)\cap N$, ,

(ii) $\varphi(rst)\cup M \supseteq \varphi(t)\cap N$,

$(\varphi(rst)\cup M \sqsupseteq \psi(r)\cap N$

$\varphi(rst)\cup M \sqsupseteq \varphi(s)\cap N\ \forall\ r, s, t\in S$ It will be denoted by $\varphi^t_{[M,N]SI\text{-}L_i}$ ($\varphi^t_{[M,N]SI\text{-}R_i}$, $\varphi^t_{[M,N]SI\text{-}E_i}$).

(φ, S, U) is called (M, N) soft intersectional two-sided ideal of S, if it is $\varphi^t_{[M,N]SI\text{-}L_i}$ and $\varphi^t_{[M,N]SI\text{-}R_i}$ of S and denoted by $\varphi^t_{[M,N]SI\text{-}T_i}$.

(φ, S, U) is called (M, N) soft intersectional ideal of S, if it is $\varphi^t_{[M,N]SI\text{-}T_i}$ and $\varphi^t_{[M,N]SI\text{-}E_i}$ of S and denoted by $\varphi^t_{[M,N]SI\text{-}I}$.

If $0\in S$. Then clearly, $\varphi(0)\cup M \supseteq \varphi(s)\cap N$ and $(\varphi(0)\cap N)\cup M \supseteq (\varphi(s)\cap N)\cup M$, $\forall\ s\in S$.

3.5 Definition

For two soft intersectional sets $(\varphi_1, ,SU)$ and $(\varphi_2, ,SU)$,

$$\varphi_1 \subseteq_{[X,Y]} \varphi_2 \Leftrightarrow (\varphi(s)\cap N)\cup M \supseteq (\varphi(s)\cap N)\cup M,$$
$\forall\ s\in S.$

Obviously, $\varphi_1 =_{[M,N]} \varphi_2 \Leftrightarrow \varphi_1 \subseteq_{[M,N]} \varphi_2$ and $\varphi_2 \subseteq_{[M,N]} \varphi_1$.

3.6 Theorem

(φ, S, U) is $\varphi^t_{[M,N]SI\text{-}S}$ of S if and only if

$$\varphi+\varphi \subseteq_{[M,N]} \varphi \text{ and } \varphi^3 \subseteq_{[M,N]} \varphi.$$

Proof: Let us assume (φ, S, U) is an $\varphi^t_{[M,N]SI\text{-}S}$ of S. Then $\forall\ s\in S$

$$((\varphi+\varphi)(s)\cap N)\cup M$$
$$= \bigcup_{s=m+n} [\varphi(m)\cap\varphi(n)\cap N]\cup M$$
$$= \bigcup_{s=m+n} [(\varphi(m)\cap\varphi(n)\cap N)\cap N]\cup M$$
$$\subseteq \bigcup_{s=m+n} [(\varphi(m+n)\cup M)\cap N]\cup M$$
$$= \bigcup_{s=m+n} [\varphi(s)\cap N]\cup M$$
$$= (\varphi(s)\cap N)\cup M\,.$$

Thus, $\varphi+\varphi\subseteq_{[M,N]}\varphi$.

$\varphi^3\subseteq_{[M,N]}\varphi$ is analogous.

Conversely, let us assume $\varphi+\varphi\subseteq_{[M,N]}\varphi$ and $\varphi^3\subseteq_{[M,N]}\varphi$. Then $\forall\ r,\ s\in S$

$$(\varphi(r+s)\cap N)\cup M\supseteq((\varphi+\varphi)(r+s)\cap N)\cup M$$
$$=\bigcup_{r+s=m+n}[\varphi(m)\cap\varphi(n)\cap N]\cup M$$
$$\supseteq(\varphi(r)\cap\varphi(s)\cap N)\cup M.$$

In the same pattern, one can prove

$$(\varphi(rst)\cap N)\cup M\supseteq(\varphi(r)\cap\varphi(s)\cap\varphi(t)\cap N)\cup M$$

Thus, (φ,S,U) is an $\underset{[M,N]SI\text{-}S}{\varphi^t}$ of S.

3.7 Lemma

(φ,S,U) is an $\underset{[M,N]SI\text{-}L_i}{\varphi^t}$ ($\underset{[M,N]SI\text{-}R_i}{\varphi^t}$, $\underset{[M,N]SI\text{-}E_i}{\varphi^t}$) of S if and only if $\varphi+\varphi\subseteq_{[M,N]}\varphi$ and $C_SC_S\varphi\subseteq_{[M,N]}\varphi$ ($\varphi C_SC_S\subseteq_{[M,N]}\varphi$, $C_S\varphi C_S\subseteq_{[M,N]}\varphi$).

Proof: Let us assume (φ,S,U) be an $\underset{[M,N]SI\text{-}L_i}{\varphi^t}$ of S.

Then, by Theorem **3.6**, we have $\varphi+\varphi\subseteq_{[M,N]}\varphi$ $\forall\ r\in S$.

Next, $((C_SC_S\varphi)(r)\cap N)\cup M$

$$=\bigcup_{r=\sum_{i=1}^{n}m_in_io_i}\{\cap_i[C_S(m_i)\cap C_S(n_i)\cap\varphi(o_i)]\cap N\}\cup M$$
$$\subseteq\bigcup_{r=\sum_{i=1}^{n}m_in_io_i}\{\cap_i[(U\cap N)\cap(\varphi(\textstyle\sum_{i=1}^{n}m_in_io_i)\cap N)]$$
$$\cap N\}\cup M$$
$$\subseteq\bigcup_{r=\sum_{i=1}^{n}m_in_io_i}(\varphi(r)\cap N)\cup M$$
$$=(\varphi(r)\cap N)\cup M.$$

Thus, $C_SC_S\varphi\subseteq_{[M,N]}\varphi$.

Conversely, let us assume $\varphi+\varphi\subseteq_{[M,N]}\varphi$ and $C_SC_S\varphi\subseteq_{[M,N]}\varphi$. Then, by Theorem **3.6**, we have

$$(\varphi(r+t)\cap N)\cup M\supseteq(\varphi(r)\cap\varphi(t)\cap N)\cup M$$

$\forall\ r,t\in S$.

Next,

$$(\varphi(rst)\cap N)\cup M$$
$$\supseteq((C_SC_S\varphi)(rst)\cap N)\cup M$$
$$=\bigcup_{rst=\sum_{i=1}^{n}m_in_io_i}\{\cap_i[C_S(m_i)\cap C_S(n_i)\cap\varphi(o_i)]$$
$$\cap N\}\cup M$$
$$\supseteq[C_S(r)\cap C_S(s)\cap\psi(t)\cap N]\cup M$$
$$=[U\cap U\cap\varphi(t)\cap N]\cup M$$
$$=(\varphi(t)\cap N)\cup M$$

Hence, ψ be a $\underset{[M,N]SI\text{-}L_i}{\varphi^t}$ of S.

3.8 Theorem

$\Phi \neq W \subseteq S$ is a ternary semiring of S if and only if $C_S(r)$ is an $\varphi^t_{[M,N,]SI\text{ }S\text{-}}$ of S.

Proof: Suppose that W is a ternary semiring of S and $r,s,t \in S$.

Case-I:

If $r,s,t \in W$, then $r+t, rst \in W$. Then

$$(C_W(r+t) \cap N) \cup M$$
$$= (U \cap N) \cup M$$
$$= (U \cap U \cap N) \cup M$$
$$= (C_W(r) \cap C_W(t) \cap N) \cup M \text{ and}$$
$$(C_W(rst) \cap N) \cup M$$
$$= (U \cap N) \cup M$$
$$= (U \cap U \cap U \cap N) \cup M$$
$$= (C_W(r) \cap C_W(s) \cap C_W(t) \cap N) \cup M.$$

Case-II:

If at least one, say $t \notin W$, then $C_W(t) = \Phi$.

Then $(C_W(r+t) \cap N) \cup M$

$$\supseteq (\Phi \cap N) \cup M$$
$$= (C_W(r) \cap \Phi \cap N) \cup M$$
$$= C_W(r) \cap C_W(t) \cap N,$$

and $(C_W(rst) \cap N) \cup M$

$$\supseteq (\Phi \cap N) \cup M$$
$$= (C_W(r) \cap \Phi \cap C_W(t) \cap N) \cup M$$
$$= (C_W(r) \cap C_W(s) \cap C_W(t) \cap N) \cup M.$$

By combining both cases

$$(C_W(r+t) \cap N) \cup M \supseteq (C_W(r) \cap C_W(t) \cap N) \cup M$$

and

$$(C_W(rst) \cap N) \cup M$$
$$\supseteq (C_W(r) \cap C_W(s) \cap C_W(t) \cap N) \cup M.$$

Hence, C_W is an $\varphi^t_{[M,N]SI\text{-}S}$ of S.

Conversely, assume that C_W is an $\varphi^t_{[M,N]SI\text{-}S}$ of S and $r,s,t \in W$. Then

$$(C_W(r+t) \cap N) \cup M$$
$$\supseteq (C_W(r) \cap C_W(t) \cap N) \cup M$$
$$= (U \cap U \cap N) \cup M = (U \cap N) \cup M.$$
$$\Rightarrow C_W(r+t) = U,$$

and

$$(C_W(rst) \cap N) \cup M$$
$$\supseteq (C_W(r) \cap C_W(s) \cap C_W(t) \cap N) \cup M$$
$$= (U \cap U \cap U \cap N) \cup M = (U \cap N) \cup M.$$
$$\Rightarrow C_W(rst) = U.$$

Thus, $r+t,\ rst \in W,\ \forall\ r,\ s,\ t \in W$.

This shows that W is a $\varphi^t_{[M,N]SI\text{-}S}$ of S.

3.9 Theorem

Let $\Phi \neq W \subseteq S$. Then *W* is a left (right, letral) ideal of *S* if and only if C_W is an $\varphi^t_{[M,N]_{SI\text{-}L_i}}$ ($\varphi^t_{[M,N]_{SI\text{-}R_i}}$, $\varphi^t_{[M,N]_{SI\text{-}E_i}}$) of S.
of *S*.

Proof: Let *W* be a ternary left ideal of *S* and $r,s,t \in S$.
Case-I: For $r,t \in W$ and $t \in W$, we have
$r+t \in W$ and $rst \in W$, respectively.
Then $(C_W(r+t) \cap N) \cup M$
$= (U \cap N) \cup M = (U \cap U \cap N) \cup M$
$= (C_W(r) \cap C_W(t) \cap N) \cup M$,
and $(C_W(rst) \cap N) \cup M$
$= (U \cap N) \cup M$
$= (C_W(t) \cap N) \cup M$.
Case-II: If $t \notin W$, then $r+t \notin W$ and $rst \notin W$.
Then $(C_W(r+t) \cap N) \cup M$
$\supseteq (\Phi \cap N) \cup M$
$= (C_W(r) \cap \Phi \cap N) \cup M$
$= (C_W(r) \cap C_W(t) \cap N) \cup M$,
and $(C_W(rst) \cap N) \cup M$
$\supseteq (\Phi \cap N) \cup M$
$= (C_W(t) \cap N) \cup M$.
By combining the both cases, we have
$(C_W(r+t) \cap N) \cup M \supseteq (C_W(r) \cap C_W(t) \cap N) \cup M$
and
$(C_W(rst) \cap N) \cup M \supseteq (C_W(t) \cap N) \cup M$.
Hence, C_W is $\varphi^t_{[M,N]_{SI\text{-}L_i}}$ of S.

Conversely, assume that C_W is an $\varphi^t_{[M,N]_{SI\text{-}L_i}}$ of S
and $r,s,t \in W$.
Then $(C_W(v+w) \cap Y) \cup X$
$\supseteq (C_W(v) \cap C_W(w) \cap Y) \cup X$
$= (U \cap U \cap Y) \cup X = (U \cap Y) \cup$,
and $(C_W(rst) \cap N) \cup M$
$\supseteq (C_W(t) \cap N) \cup M$
$= (U \cap N) \cup M$.
Thus, $r+t \in W$ and $rst \in W$, $\forall\, r,t \in W$ and
$\forall\, t \in W$, respectively.
This shows that, W is ternary left ideal of S.

3.10 Theorem

Let (φ_1, S, U), (φ_2, S, U) be the $\underset{[M,N]}{\varphi_i^t}{}_{SI\text{-}L_i}$'s ($\underset{[M,N]}{\varphi_i^t}{}_{SI\text{-}R_i}$'s , $\underset{[M,N]}{\varphi_i^t}{}_{SI\text{-}E_i}$'s) of S (where i=1, 2), then their sum $(\varphi_1+\varphi_2, S, U)$ is also an $\underset{[M,N]}{(\varphi_1+\varphi_2)^t}{}_{SI\text{-}L_i}$ ($\underset{[M,N]}{(\varphi_1+\varphi_2)^t}{}_{SI\text{-}R_i}$, $\underset{[M,N]}{(\varphi_1+\varphi_2)^t}{}_{SI\text{-}E_i}$) of S.

Proof: To show that $(\varphi_1+\varphi_2, S, U)$ is an $\underset{[M,N]}{(\varphi_1+\varphi_2)^t}{}_{SI\text{-}L_i}$ of S, we will have to prove that [M N,]

(i) $((\varphi_1+\varphi_2)(r+t)\cap N)\cup M$
$\supseteq((\varphi_1+\varphi_2)(r)\cap(\varphi_1+\varphi_2)(t)\cap N)\cup M$
$\forall\, r,t\in S.$

(ii) $((\varphi_1+\varphi_2)(rst)\cap N)\cup M$
$\supseteq((\varphi_1+\varphi_2)(t)\cap N)\cup M \quad \forall\, r,s,t\in S.$

Now,

$$\begin{aligned}
&((\varphi_1+\varphi_2)(r)\cap(\varphi_1+\varphi_2)(t)\cap N)\cup M\\
&=(\{\bigcup_{r=m+n}[\varphi_1(m)\cap\varphi_2(n)]\}\cap\{\bigcup_{t=p+q}[\varphi_1(p)\cap\varphi_2(q)]\}\\
&\quad\cap N)\cup M\\
&=\bigcup_{r=m+n,\,t=p+q}[\varphi_1(m)\cap\varphi_2(n)\cap\varphi_1(p)\cap\varphi_2(q)\cap N]\cup M\\
&=\bigcup_{r=m+n,\,t=p+q}[(\varphi_1(m)\cap\varphi_1(p)\cap N)\cap\\
&\qquad(\varphi_2(n)\cap\varphi_2(q)\cap N)\cap N]\cup M\\
&\subseteq\bigcup_{r+t=m+p+n+q}[\varphi_1(m+p)\cap\varphi_2(n+q)\cap N]\cup M\\
&\subseteq\bigcup_{r+t=g'+h'}[\varphi_1(g')\cap\varphi_2(h')\cap N]\cup M\\
&=((\varphi_1+\varphi_2)(r+t)\cap N)\cup M,
\end{aligned}$$

and $((\varphi_1+\varphi_2)(t)\cap N)\cup M$

$$\begin{aligned}
&=\bigcup_{t=u+v}[\varphi_1(u)\cap\varphi_2(v)\cap N]\cup M\\
&=\bigcup_{t=u+v}[(\varphi_1(u)\cap\varphi_2(v)\cap N)\cap N]\cup M\\
&\subseteq\bigcup_{rst=rsu+rsv}[\varphi_1(rsu)\cap\varphi_2(rsv)\cap N]\cup M\\
&\subseteq\bigcup_{rst=u'+v'}[\varphi_1(u')\cap\varphi_2(v')\cap N]\cup M\\
&=((\varphi_1+\varphi_2)(rst)\cap N)\cup M.
\end{aligned}$$

Hence, $(\varphi_1+\varphi_2, S, U)$ is an $\underset{[M,N]}{(\varphi_1+\varphi_2)^t}{}_{SI\text{-}L_i}$ of S.

3.11 Theorem

Let (φ_1,S,U), (φ_2,S,U) and (φ_3,S,U) be the $\varphi^t_{i\,[M,N]SI\text{-}L_i\text{'s}}$ ($\varphi^t_{i\,[M,N]SI\text{-}R_i\text{'s}}$, $\varphi^t_{i\,[M,N]SI\text{-}E_i\text{'s}}$) of S (where i=1, 2, 3), then $(\varphi_1\varphi_2\varphi_3,S,U)$ is also an $(\varphi_1\varphi_2\varphi_3)^t_{\underset{[M,N]}{SI\text{-}L_i}}$ ($(\varphi_1\varphi_2\varphi_3)^t_{\underset{[M,N]}{SI\text{-}R_i}}$, $(\varphi_1\varphi_2\varphi_3)^t_{\underset{[M,N]}{SI\text{-}E_i}}$) of S.

Proof: Let r, $t\in S$. Then

$$(\varphi_1\varphi_2\varphi_3)(r)=\bigcup_{r=\sum_{i=1}^{k}m_in_io_i}\{\cap_i[\varphi_1(m_i)\cap\varphi_2(n_i)\cap\varphi_3(o_i)]\}$$

$$(\varphi_1\varphi_2\varphi_3)(t)=\bigcup_{t=\sum_{j=1}^{l}m'_jn'_jo'_j}\{\cap_j[\varphi_1(m'_j)\cap\varphi_2(n'_j)\cap\varphi_3(o'_j)]\}$$

Now,

$$\big((\varphi_1\varphi_2\varphi_3)(r)\cap(\varphi_1\varphi_2\varphi_3)(t)\cap N\big)\cup M$$
$$=([\bigcup_{r=\sum_{i=1}^{k}m_in_io_i}\{\cap_i[\varphi_1(m_i)\cap\varphi_2(n_i)\cap\varphi_3(o_i)]\}]\cap[\bigcup_{t=\sum_{j=1}^{l}m'_jn'_jo'_j}\{\cap_j[\varphi_1(m'_j)\cap\varphi_2(n'_j)\cap\varphi_3(o'_j)]\}]\cap N)\cup M$$

$$=\bigcup_{r=\sum_{i=1}^{k}m_in_io_i,\ t=\sum_{j=1}^{l}m'_jn'_jo'_j}[\cap_i\cap_j[\varphi_1(m_i)\cap\varphi_2(n_i)\cap\varphi_3(o_i)\cap\varphi_1(m'_j)\cap\varphi_2(n'_j)\cap\varphi_3(o'_j)]\cap N]\cup M$$

$$=\bigcup_{r+t=\sum_{i=1}^{k}m_in_io_i+\sum_{j=1}^{l}m'_jn'_jo'_j}[\cap_i\cap_j[\big(\varphi_1(m_i)\cap\varphi_1(m'_j)\cap N\big)\cap\big(\varphi_2(n_i)\cap\varphi_2(n'_j)\cap N\big)\cap\big(\varphi_3(o_i)\cap\varphi_3(o'_j)\cap N\big)]\cap N]\cup M$$

$$\subseteq\bigcup_{r+t=\sum_{p=1}^{q}x_py_pz_p}\{\cap_p[\varphi_1(x_p)\cap\varphi_2(y_p)\cap\varphi_3(z_p)]\cap N\}\cup M$$

$$=((\varphi_1\varphi_2\varphi_3)(r+t)\cap N)\cup M.$$

Next, for $r,s,t\in S$

$$((\varphi_1\varphi_2\varphi_3)(t)\cap N)\cup M$$
$$=\bigcup_{t=\sum_{i=1}^{k}x_iy_iz_i}\{\cap_i[\varphi_1(x_i)\cap\varphi_2(y_i)\cap\varphi_3(z_i)]\cap Y\}\cup X$$
$$=\bigcup_{rst=\sum_{i=1}^{k}rsx_iy_iz_i}\{\cap_i[\varphi_1(rsx_i)\cap\varphi_2(y_i)\cap\varphi_3(z_i)]\cap N\}\cup M$$

$\subseteq \bigcup_{rst=\sum_{i=1}^{k} x_i' y_i' z_i'} \{\bigcap_i [\varphi_1(x_i') \cap \varphi_2(y_i') \cap \varphi_3(z_i')] \cap N\} \cup M$

$= ((\varphi_1\varphi_2\varphi_3)(rst) \cap N) \cup M$.

Thus, $((\varphi_1\varphi_2\varphi_3)(r+t) \cap N) \cup M$

$\supseteq ((\varphi_1\varphi_2\varphi_3)(r) \cap (\varphi_1\varphi_2\varphi_3)(t) \cap N) \cup M$,

$\forall\ r$, $t \in S$ and $((\varphi_1\varphi_2\varphi_3)(rst) \cap N) \cup M$

$\supseteq ((\varphi_1\varphi_2\varphi_3)(t) \cap N) \cup M$, $\forall\ r,s,t \in S.$

Therefore, $(\varphi_1\varphi_2\varphi_3, S, U)$ is an $(\varphi_1\varphi_2\varphi_3)^t_{\underset{[M,N]}{SI\text{-}L_i}}$ of S.

3.12 Theorem

For a ternary semiring *S*, the following are equivalent

(i) S is Ŧ $_{vN\ r\text{-}}$.

(ii) $E \cap F \cap G = EFG$,, for any $E\ F$, and G as Ŧ $_{R}i$, Ŧ $_{E}i$ and Ŧ $_{L}i$ of S, respectively.

(iii) $\varphi_1 \cap \varphi_2 \cap \varphi_3 = \varphi_1\varphi_2\varphi_3$, for any φ_1, φ_2 and φ_3 as $\underset{[M,N]SI\text{-}R_i}{\varphi_1^{\ t}}$, $\underset{[M,N]SI\text{-}E_i}{\varphi_2^{\ t}}$ and $\underset{[M,N]SI\text{-}L_i}{\varphi_3^{\ t}}$ of S, respectively.

Proof: $(i) \Rightarrow (iii)$ Let φ_1, φ_2 and φ_3 are $\underset{[M,N]SI\text{-}R_i}{\varphi_1^{\ t}}$, $\underset{[M,N]SI\text{-}E_i}{\varphi_2^{\ t}}$ and $\underset{[M,N]SI\text{-}L_i}{\varphi_3^{\ t}}$ of S, respectively. Then for any $w \in S$, we have

$((\varphi_1\varphi_2\varphi_3)(t) \cap N) \cup M$

$= \bigcup_{t=\sum_{i=1}^{k} m_i n_i o_i} \{\bigcap_i [\varphi_1(m_i) \cap \varphi_2(n_i) \cap \varphi_3(o_i)] \cap N\} \cup M$

$\subseteq \bigcup_{t=\sum_{i=1}^{k} m_i n_i o_i} \{\bigcap_i [\varphi_1(m_i n_i o_i) \cap \varphi_2(m_i n_i o_i) \cap$

$\varphi_3(m_i n_i o_i)] \cap N\} \cup M$

$\subseteq \bigcup_{t=\sum_{i=1}^{k} m_i n_i o_i} \{\bigcap_i [(\varphi_1(m_i n_i o_i) \cap N) \cap$

$(\varphi_2(m_i n_i o_i) \cap N) \cap (\varphi_3(m_i n_i o_i) \cap N)]$

$\cap N\} \cup M$

$= \bigcup_{t=\sum_{i=1}^{k} m_i n_i o_i} [\{\bigcap_i \varphi_1(m_i n_i o_i)\} \cap \{\bigcap_i \varphi_2(m_i n_i o_i)\}$

$\cap \{\bigcap_i \varphi_3\ (m_i n_i o_i)\} \cap\ N] \cup\ M$

$\subseteq \bigcup_{t=\sum_{i=1}^{k} m_i n_i o_i} [\{\varphi_1(\sum_{i=1}^{k} m_i n_i o_i) \cap \varphi_2(\sum_{i=1}^{k} m_i n_i o_i)$

$\cap \varphi_3(\sum_{i=1}^{k} m_i n_i o_i)\} \cap\ N] \cup\ M$

$= \bigcup_{t=\sum_{i=1}^{k} m_i n_i o_i} \{\varphi_1(t) \cap \varphi_2(t) \cap \varphi_3(t) \cap N\} \cup M$

$= (\varphi_1(t) \cap \varphi_2(t) \cap \psi_3(t) \cap N) \cup M$

$= ((\varphi_1 \cap \varphi_2 \cap \varphi_3)(t) \cap N) \cup M.$

Thus,

$(\varphi_1\varphi_2\varphi_3 \cap N) \cup M \subseteq (\varphi_1 \cap \varphi_2 \cap \varphi_3 \cap N) \cup M$

Since, S is Ŧ$_{vN\text{-}r}$. Then for any $s \in S$, there exist $p \in S$ such that $s = sps$.

Now,

$((\varphi_1 \cap \varphi_2 \cap \varphi_3)(s) \cap N) \cup M$

$= (\varphi_1(s) \cap \varphi_2(s) \cap \varphi_3(s) \cap N) \cup M$

$= (\varphi_1(s) \cap (\varphi_2(s) \cap Y) \cap \varphi_3(s) \cap N) \cup M$

$\subseteq (\varphi_1(s) \cap \varphi_2(sps) \cap \varphi_3(s) \cap N) \cup M.$

Thus,

$((\varphi_1 \cap \varphi_2 \cap \varphi_3)(s) \cap N) \cup M$

$\subseteq \bigcup_{w=\sum_{i=1}^{n} x_i y_i z_i} \{\bigcap_i [\varphi_1(x_i) \cap \varphi_2(y_i) \cap \varphi_3(z_i)] \cap N\} \cup M$

$= ((\varphi_1\varphi_2\varphi_3)(s) \cap N) \cup M.$

This implies

$(\varphi_1 \cap \varphi_2 \cap \varphi_3 \cap N) \cup M \subseteq (\varphi_1\varphi_2\varphi_3 \cap N) \cup M$

Hence, finally we get

$\varphi_1 \cap \varphi_2 \cap \varphi_3 =_{[M,N]} \varphi_1\varphi_2\varphi_3.$

$(iii) \Rightarrow (ii)$ Assume (iii) holds. Now, let E, F and G are Ŧ$_{R_i}$, Ŧ$_{E_i}$ and Ŧ$_{L_i}$ of S, respectively.

Then, the characteristic functions C_E, C_F and C_G are $\underset{[M,N]SI\text{-}R_i}{\varphi^t}$, $\underset{[M,N]SI\text{-}E_i}{\varphi^t}$ and $\underset{[M,N]SI\text{-}L_i}{\varphi^t}$ of S, respectively. Next, by hypothesis

$C_{EFG} =_{[M,N]} C_E \cdot C_F \cdot C_G =_{[M,N]} C_E \cap C_F \cap C_G$

$\Rightarrow C_{EFG} =_{[M,N]} C_{E \cap F \cap G}$

$\Rightarrow EFG = E \cap F \cap G.$

$(i) \Leftrightarrow (ii)$ is followed by Theorem **2.7**.

3.13 Theorem

For a ternary semiring S with 1, the following are equivalent:

(i) S is Ŧ$_{R_{w\text{-}r}}$.

(ii) All Ŧ$_{R_i\text{'s}}$ of S are idempotent.

(iii) $\varphi_1 \cap \varphi_2 \cap \varphi_3 = \varphi_1\varphi_2\varphi_3$, for any φ_1, φ_2 and φ_3 as $\underset{[M,N]SI\text{-}R_i}{\varphi_1^t}$, $\underset{[M,N]SI\text{-}E_i}{\varphi_2^t}$ and $\underset{[M,N]SI\text{-}L_i}{\varphi_3^t}$ of S, respectively.

(iv) All $\underset{[M,N]SI\text{-}R_i\text{'s}}{\varphi^t}$ of S are fully idempotent.

(v) $\varphi_1 \cap \varphi_2 \cap \varphi_3 = \varphi_1\varphi_2\varphi_3$, for any φ_1, φ_2 and φ_3 as $\underset{[M,N]SI\text{-}R_i}{\varphi_1{}^t}$, $\underset{[M,N]SI\text{-}E_i}{\varphi_2{}^t}$ and $\underset{[M,N]SI\text{-}L_i}{\varphi_3{}^t}$ of S, respectively.

If S is commutative then (i)-(v) are equivalent to

(vi) S is $\text{Ŧ}_{vN\text{-}r}$.

Proof: $(i) \Rightarrow (iv)$ Let φ be an $\underset{[M,N]SI\text{-}R_i}{\varphi^t}$ of S and $r \in S$. Then

$$(\varphi^3(r) \cap N) \cup M$$
$$= ((\varphi\varphi\varphi)(r) \cap N) \cup M$$
$$= \bigcup_{r=\sum_{i=1}^k m_i n_i o_i} \{\textstyle\bigcap_i [\varphi(m_i) \cap \varphi(n_i) \cap \varphi(o_i)] \cap N\} \cup M$$
$$= \bigcup_{r=\sum_{i=1}^k m_i n_i o_i} \{\textstyle\bigcap_i [(\varphi(m_i) \cap N) \cap \varphi(n_i) \cap \varphi(o_i)] \cap N\} \cup M$$
$$\subseteq \bigcup_{r=\sum_{i=1}^k m_i n_i o_i} \{\textstyle\bigcap_i [\varphi(m_i n_i o_i) \cap \varphi(n_i) \cap \varphi(o_i)] \cap N\} \cup M$$
$$= \bigcup_{r=\sum_{i=1}^k m_i n_i o_i} [\{\textstyle\bigcap_i \varphi_1(m_i n_i o_i)\} \cap \{\bigcap_i (\varphi_2(n_i) \cap \varphi_3(o_i)\}) \cap N] \cup M$$
$$= \bigcup_{r=\sum_{i=1}^k m_i n_i o_i} [\{\varphi_1(\textstyle\sum_{i=1}^k m_i n_i o_i)\} \cap \{\bigcap_i (\varphi_2(n_i) \cap \varphi_3(o_i))\} \cap N] \cup M$$
$$\subseteq \bigcup_{r=\sum_{i=1}^k m_i n_i o_i} [\varphi(r) \cap \{\textstyle\bigcap_i (\varphi_2(n_i) \cap \varphi_3(o_i))\} \cap N] \cup M$$
$$\subseteq \bigcup_{r=\sum_{i=1}^k m_i n_i o_i} [\varphi(r) \cap N] \cup M$$
$$= (\varphi(r) \cap N) \cup M.$$

This implies $\varphi^3 \subseteq_{[M,N]} \varphi$.

Since, S is $\text{Ŧ}_{R_{w\text{-}r}}$. So, $r \in rSrSrSS$ and we can write $r = \sum_{i=1}^k r m_i r n_i r p_i q_i$, where $m_i, n_i, p_i, q_i \in S$ and $k \in \mathbb{N}$.

Then

$$(\varphi(r) \cap \varphi(r) \cap \varphi(r) \cap N) \cup M$$
$$\subseteq (\varphi(rm_i r) \cap \varphi(rn_i r) \cap \varphi(rp_i q_i) \cap N) \cup M, \ \forall\, i.$$
$$\Rightarrow (\varphi(r) \cap N) \cup M$$
$$\subseteq \textstyle\bigcap_i \{\varphi(rm_i r) \cap \varphi(rn_i r) \cap \varphi(rp_i q_i) \cap N\} \cup M$$

$$\subseteq \bigcup_{r=\sum_{i=1}^{k} rm_i r(rn_i r) rp_i q_i} \{\cap_i[\varphi(rm_i r)\cap\varphi(rn_i r)$$
$$\cap\varphi(rp_i q_i)]\cap N\}\cup M$$
$$\subseteq \bigcup_{r=\sum_{j=1}^{l} a'_j b'_j c'_j} \{\cap_j[\varphi(a'_j)\cap\varphi(b'_j)\cap\varphi(c'_j)]$$
$$\cap N\}\cup M$$
$$=((\varphi\varphi\varphi)(r)\cap N)\cup M$$
$$=\left(\varphi^3\cap N\right)\cup M.$$

This implies $\varphi \subseteq_{[M,N]} \varphi^3$.

Thus, $\varphi =_{[M,N]} \varphi^3$. Therefore, φ is fully idempotent.

$(iv)\Rightarrow(i)$ Let $r\in S$ and $V=rSS$ be a Ŧ_{R_i} of S, formed by r. Then $r\in V$ and the characteristic function C_V of V is an $\underset{[M,N]_{SI\text{-}R_i}}{\varphi^t}$ of S. Next, by hypothesis $C_V =_{[M,N]} C_V\cdot C_V\cdot C_V =_{[M,N]} C_{V^3}$

$$\Rightarrow V=V^3$$
$$\Rightarrow r\in V^3=(rSS)^3$$
$$\Rightarrow r\in rSrSrSS.$$

Thus, S is $\text{Ŧ}_{R_{w\text{-}r}}$.

$(i)\Rightarrow(v)$ Let φ_1, φ_2 and φ_3 as $\underset{[M,N]_{SI\text{-}R_i}}{\varphi_1{}^t}$, $\underset{[M,N]_{SI\text{-}E_i}}{\varphi_2{}^t}$ and $\underset{[M,N]_{SI\text{-}L_i}}{\varphi_3{}^t}$ of S, respectively. Then for any $r\in S$, we have

$$((\varphi_1\varphi_2\varphi_3)(r)\cap N)\cup M$$
$$=\bigcup_{r=\sum_{i=1}^{k} x_i y_i z_i} \{\cap_i[\varphi_1(x_i)\cap\varphi_2(y_i)\cap\varphi_3(z_i)]$$
$$\cap N\}\cup M$$
$$=\bigcup_{r=\sum_{i=1}^{k} x_i y_i z_i} \{\cap_i[\varphi_1(x_i)\cap\varphi_2(y_i)\cap\varphi_3(z_i)]$$
$$\cap N\}\cup M$$
$$=\bigcup_{r=\sum_{i=1}^{k} x_i y_i z_i} \{\cap_i[(\varphi_1(x_i)\cap N)\cap(\varphi_2(y_i)\cap N)$$
$$\cap(\varphi_3(z_i)\cap N)]\cap N\}\cup M$$
$$=\bigcup_{r=\sum_{i=1}^{k} x_i y_i z_i} \{\cap_i[\varphi_1(x_i y_i z_i)\cap\varphi_2(x_i y_i z_i)$$
$$\cap\varphi_3(x_i y_i z_i)]\cap N\}\cup M$$
$$=\bigcup_{r=\sum_{i=1}^{k} x_i y_i z_i} [\{\cap_i(\varphi_1(x_i y_i z_i))\}\cap\{\cap_i(\varphi_2(x_i y_i z_i))\}$$
$$\cap\{\cap_i(\varphi_3(x_i y_i z_i))\}\cap N]\cup M$$

$= \bigcup_{r=\sum_{i=1}^{k} x_i y_i z_i} [\{\varphi_1(\sum_{i=1}^{k} x_i y_i z_i)\} \cap \{\varphi_2(\sum_{i=1}^{k} x_i y_i z_i)\}$

$\cap \{\varphi_3(\sum_{i=1}^{k} x_i y_i z_i)\} \cap N] \cup M$

$= \bigcup_{r=\sum_{i=1}^{k} x_i y_i z_i} [\varphi_1(r) \cap \varphi_2(r) \cap \varphi_3(r) \cap N] \cup M$

$= (\varphi_1(r) \cap \varphi_2(r) \cap \varphi_3(r) \cap N) \cup M$

$= ((\varphi_1 \cap \varphi_2 \cap \varphi_3)(r) \cap N) \cup M.$

This implies $\varphi_1\varphi_2\varphi_3 \subseteq_{[M,N]} \varphi_1 \cap \varphi_2 \cap \varphi_3$.

Since, S is Ŧ $_{R_{w\text{-}r}}$. Then $r \in S$ can be written as

$r = \sum_{i=1}^{k} rm_i rn_i rp_i q_i$ where $m_i, n_i, p_i, q_i \in S$ and

$k \in N$.

Now,

$((\varphi_1 \cap \varphi_2 \cap \varphi_3)(r) \cap N) \cup M$

$= (\varphi_1(r) \cap \varphi_2(r) \cap \varphi_3(r) \cap N) \cup M$

$\subseteq (\varphi_1(rm_i r) \cap \varphi_2(rn_i r) \cap \varphi_3(rp_i q_i) \cap N) \cup M$

Thus, we have

$\subseteq ((\varphi_1 \cap \varphi_2 \cap \varphi_3)(r) \cap N) \cup M$

$\subseteq \bigcap_i \{\varphi_1(rm_i r) \cap \varphi_2(rn_i r) \cap \varphi_3(rp_i q_i)$

$\cap N\} \cup M$

$\subseteq \bigcup_{r=\sum_{i=1}^{k} rm_i r (rn_i r) rp_i q_i} \{\bigcap_i [\varphi_1(rm_i r) \cap \varphi_2(rn_i r)$

$\cap \varphi_3(rp_i q_i)] \cap N\} \cup M$

$\subseteq \bigcup_{r=\sum_{j=1}^{k} a'_j b'_j c'_j} \{\bigcap_j [\varphi_1(a'_j) \cap \varphi_2(b'_j) \cap \varphi_3(c'_j)]$

$\cap Y\} \cup X$

$= ((\varphi_1\varphi_2\varphi_3)(r) \cap N) \cup M.$

$\Rightarrow \varphi_1\varphi_2\varphi_3 \subseteq_{[M,N]} \varphi_1 \cap \varphi_2 \cap \varphi_3.$

Therefore, $\varphi_1 \cap \varphi_2 \cap \varphi_3 =_{[M,N]} \varphi_1\varphi_2\varphi_3$.

$(v) \Rightarrow (iii)$ Let E, F and G be the Ŧ $_{R_i}$, Ŧ $_{E_i}$ and Ŧ $_{L_i}$ of S, respectively. Then, the characteristic functions C_E, C_F and C_G are $\varphi^t_{[M,N]SI\text{-}R_i}$, $\varphi^t_{[M,N]SI\text{-}E_i}$ and $\varphi^t_{[M,N]SI\text{-}L_i}$ of S, respectively. Next, by hypothesis

$C_{EFG} =_{[M,N]} C_E.C_F.C_G =_{[M,N]} C_E \cap C_F \cap C_G$

$\Rightarrow C_{EFG} =_{[M,N]} C_E \cap C_F \cap C_G$

$\Rightarrow EFG =_{[M,N]} E \cap F \cap G.$

$(i) \Rightarrow (ii) \Rightarrow (iii)$ and $(i) \Rightarrow (iv)$ are straight forward.

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