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APPLICATION OF MAPLE PROGRAM AND MATLAB CODE ON THE STABILITY ANALYSIS OF AN EXPLICIT FOURTH-STAGE SECOND-ORDER RUNGE-KUTTA METHOD

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Abtract

The essence of this paper is to analyze the stability of a derived explicit fourth-stage second-order Runge-Kutta method using MAPLE PROGRAM and MATLAB CODE. The property of stability is essential and necessary requirement for any numerical method to be termed reliable. The analysis revealed that the method is absolutely stable. The test equation $y' = \lambda y$ was applied on the method and a polynomial equation was generated. The MAPLE program was used to resolve the polynomial equation resulting to sets of real and complex roots. The MATLAB code was used to plot the stability curve which shows the region of absolute stability. Hence, the whole analysis revealed that the method is not just stable, but it is absolutely stable.

Keywords: *Stability, Explicit, Runge-Kutta Methods, MAPLE, MATLAB CODE, Polynomial equation, Stability Curve, Test Equation, Numerical Method.*

1.0INTRODUCTION

Runge-kutta methods are numerical (one-step) methods for solving initial value problems of the form:

$$y'(x) = f(x, y), \quad y(x_0) = y_0. \tag{1.1}$$

According to Butcher (2008, 2009), in Ordinary Differential Equations, initial value problems are problems with subsidiary conditions which are called initial conditions and are applicable to solving real life problems like growth and decay problems, temperature problems, falling body problems, problems in chemical engineering, electrical circuit problems, dilution problems e.t.c. For example, a person opens an account with an initial amount that accrues interest compounded continuously and assuming no additional deposits or withdrawals, we can transform this problem to an initial-value first-order ordinary differential equation and solve for the amount that will be in the account after a period of time at a particular interest rate. Here, the initial amount will be the initial condition while the interest rate will be the constant of proportionality. The above problem is growth problem because it has to do with interest.

Explicit Runge-Kutta methods have proven to be one of the best methods for solving initial value problems in Ordinary Differential Equations. It has overtime been discovered that the method is subject to improvement, hence more research is still been carried out to get better efficiency and accuracy of the method. Many researchers have worked to improve on the accuracy of the method as can been seen in the work of Agbeboh (2013), Van der Houwen (2015), Brugnano et al (2019), Barletti et al (2020) and Gianluca et al (2021). This is what motivated us to work on a fourth–stage fourth-order method to find out how variation in parameters can yield further efficiency.

2.0 THE DERIVED METHOD

The Fourth Stage Second order Runge Kutta Method is seen below:

$$y_{n+1} = y_n + \frac{h_i}{36}(9k_1 + 2k_2 + 30k_3 - 5k_4)$$

$$k_1 = f(x_n, y_n)$$

$$k_2 = f\left(x_n + \frac{h_i}{4}, \quad y_n + \frac{3h_i}{28}k_1\right)$$

$$k_3 = f\left(x_n + \frac{3h_i}{4}, y_n + \frac{h_i}{140}(-283k_1 + 168k_2)\right)$$

$$k_4 = f\left(x_n + h_i, y_n + \frac{h_i}{35}(-339k_1 + 21k_2 + 21k_3)\right)$$

3.0STABILITY FUNCTION OF THE FOURTH STAGE SECOND ORDER METHOD

Theorem 3.0: The explicit fourth-stage second-order method is absolutely stable.

Proof: Applying the test equation $y' = \lambda y$ on the above method, we have:

$$k_1 = f(x_n, y_n) = \lambda y, \qquad k_2 = f\left(x_n + \frac{h_i}{4}, \quad y_n + \frac{3h_i}{28}k_1\right) = \lambda(y_n + \frac{3h\lambda y}{28})$$

$$k_2 = \lambda y\left(1 + \frac{3\lambda h}{28}\right)$$

$$k_3 = f\left(x_n + \frac{3h_i}{4}, y_n + \frac{h_i}{140}(-283k_1 + 168k_2)\right)$$

$$k_3 = \lambda\left(y + \frac{h}{140}\left(-283\lambda y + 168\left(\lambda y + \frac{3hy\lambda^2}{28}\right)\right)\right)$$

$$k_3 = \lambda y(1 - \frac{115h\lambda}{140} + \frac{63h^2\lambda^2}{490})$$

$$k_3 = \lambda y(1 - \frac{23h\lambda}{28} + \frac{9h^2\lambda^2}{70})$$

$$k_4 = f\left(x_n + h_i, \quad y_n + \frac{h_i}{35}(-339k_1 + 21k_2 + 21k_3)\right)$$

$$k_4 = \lambda\left(y_n + \frac{h}{35}(-339\lambda y + 21(\lambda y + \frac{3hy\lambda^2}{28}) + 21(\lambda y - \frac{23hy\lambda^2}{28} + \frac{9h^2\lambda^3 y}{70})\right)$$

$$k_4 = \lambda y\left(1 - \frac{297h\lambda}{35} - \frac{420h^2\lambda^2}{980} + \frac{189h^3\lambda^3}{2450}\right),$$

$$k_4 = \lambda y\left(1 - \frac{297h\lambda}{35} - \frac{3h^2\lambda^2}{7} + \frac{27h^3\lambda^3}{350}\right)$$

Hence, $y_{n+1} - y_n = \frac{h}{36}\left[9\lambda y + 2\lambda y\left(1 + \frac{3\lambda h}{28}\right) + 30\lambda y\left(1 - \frac{23h\lambda}{28} + \frac{9h^2\lambda^2}{70}\right) - 5\lambda y\left(1 - \frac{297h\lambda}{35} - \frac{3h^2\lambda^2}{7} + \frac{27h^3\lambda^3}{350}\right)\right]$

$$y_{n+1} - y_n = \frac{\lambda y h}{36}\left[9 + 2\left(1 + \frac{3\lambda h}{28}\right) + 30\left(1 - \frac{23h\lambda}{28} + \frac{9h^2\lambda^2}{70}\right) - 5\left(1 - \frac{297h\lambda}{35} - \frac{3h^2\lambda^2}{7} + \frac{27h^3\lambda^3}{350}\right)\right]$$

$$y_{n+1} - y_n = \frac{\lambda y h}{36}[36 + 18\lambda h + 6h^2\lambda^2]$$

Dividing by y and setting $\mu = \lambda h,\ we\ have$:

$$\frac{y_{n+1} - y_n}{y_n} = \frac{\mu}{36}[36 + 18\mu + 6\mu^2]$$

$$\frac{y_{n+1}}{y_n} - 1 = \left[\mu + \frac{\mu^2}{2} + \frac{\mu^3}{6}\right]$$

$$R(z) = \frac{y_{n+1}}{y_n} = 1 + \mu + \frac{\mu^2}{2!} + \frac{\mu^3}{3!} = 0$$

Hence, we have the stability polynomial, which is resolved below using MAPLE:
Using MAPLE to solve $R(z)$, the complex roots are:
-1.59607163798332152310,
-0.70196418100833923844, -1.80733949445202185360i,
-0.70196418100833923844, +1.80733949445202185360i.

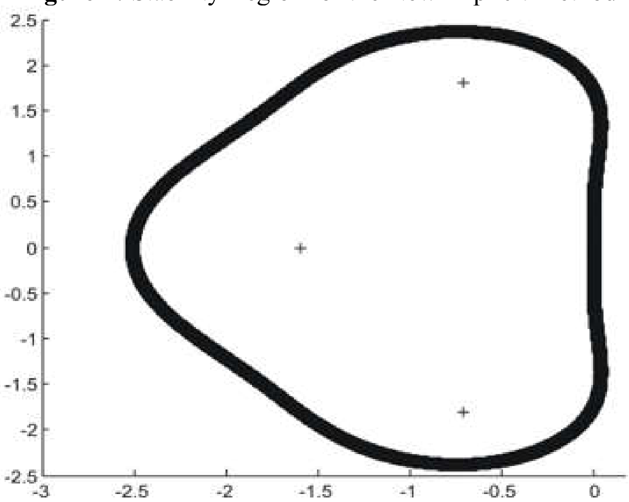
Following the definition which gives the conditions for A-stability and L- stability, it follows that $R(z)$ is said to be (i) A-stable if $|R(z)| < 1$ Whenever $Re(z) < 0$ that is z is real and negative (ii) L-stable, if it is A-stable and, in addition, satisfies
$|R(z)| \to 0$ as $Re \to -\infty$

It can be seen that the real parts of the solution of our polynomial are negative and are all less than zero, and for each real root, $|R(z)| < 1$thereby making our method to be A-stable. Also, for L-stability, we can see that the real parts of the complex roots approach $-\infty$ as$|R(z)| \to 0$ for example the absolute value of

$$1 + -1.59607163798332152310 + \frac{(-1.59607163798332152310)^2}{2} + \frac{(-1.59607163798332152310)^3}{6}$$

Gives $5.00000000000000000000\ 10^{-20}$ which tends to zero. Hence our method is also L-stable.
Plotting the complex roots on a graph (the real parts on the x-axis and the imaginary parts on the y-axis) using MATLAB CODE, we have the absolute stability region seen in the diagram below:

Figure 1: Stability Region for the New Implicit Method



5.0CONCLUSION
It is clearly seen from the figure above that the fourth stage second order explicit Runge Kutta method is A-stable and L-stable. The region of absolutely stability is also clearly seen and pointed in the above figure.

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