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ALMOST CONTRA G(b*g) * CONTINUOUS FUNCTION IN GRILL TOPOLOGICAL SPACES

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Abstract: -
The aim of the paper is to study a new class of almost contra continuous function called almost contra $G_{(b*g)*}$ continuous function. Its relation to other continuous functions are give.

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1. INTRODUCTION

It is found from literature that during the recent years many topologists are interested in the study of generalized types of closed sets. For instance, a certain form of generalized closed sets was, initiated by Levine [9].

Following this trend, we have introduced and investigated a kind of generalized closed sets the definition being formulated in terms of grills. The concept of grill was first introduced by Choquet [3] in the year 1947. From subsequent investigations it is revealed that grills can be used as an extremely useful device for investigation of a number of topological problems.

Ahmad Al-omari and Mohd-salmi Md-Noorani[2] introduced and developed almost contra continuous function E.Ekici[7] extended the notion to a class of almost contra pre continuous function T. Nolri and V. Popa[10] discussed the properties of almost contra pre continuous function.

Preliminaries

Definition 2.1: A nonempty collection G of non-empty subsets of a topological space X is called a grill if (i) $A \in G$ and $A \subseteq B \subseteq X \Longrightarrow B \in G$ and (ii) $A, B \subseteq X$ and $A \cup B \in G \Longrightarrow A \in G$ or $B \in G$.

Let G be a grill on a topological space (X, τ). In an operator $\phi: P(X) \longrightarrow P(X)$ was defined by $\phi(A) = \{ x \in X / U \cap A \in G, \forall U \in \tau(x)\}$, $\tau(x)$ denotes the neighborhood of x. Also the map $\Psi : P(X) \rightarrow P(X)$, given by $\Psi(A) = \{A \cup \phi(A)$ for all $A \in P(x)$. Corresponding to a grill G on a topological space (X, τ) there exists a unique topology τ_G on X given by $\tau_G = \{U \subseteq X / \Psi(X - U) = X - U\}$ where for any $A \subseteq X$, $\Psi(A) = A \cup \phi(A) = \tau_G -$ cl(A).

Thus a subset A of x is τ_G – closed (resp. τ_G – dense in itself) if $\psi(A) = A$ or equivalently if $\phi(A) \subseteq A$ (resp. $A \subseteq \phi(A)$).

In the next section, we introduce and analyze a new class of generalized contra continuity called almost contra $G_{(b^*g)^*}$ continuity.

Throughout the paper, by on space X we always mean a topological space (X, τ) with no separation properties assumed. If $A \subseteq X$, we shall adopt the usual notations int(A) and cl(A) respectively for the interior and closure of A in (X, τ). Again τ_G – cl(A) and τ_G – int(A) will respectively denote the closure and interior of A in (X, τ_G). Similarly, whenever we say that a subset A of a space X is open (or closed), it will mean that A is open (or closed) in (X, τ). For open and closed sets with respective to any other topology on X, eg. τ_G we shall write τ_G – open and τ_G – closed. The collection of all open neighborhoods of a point x in (X, τ) will be denoted $\tau(x)$.

(x, τ, G) denotes a topological space (X, τ) with a grill G.

Definition 2.2: A subset A of a topological space (X, τ) is called

1. b open if $A \subseteq \text{int cl}(A) \cup \text{cl}(\text{int}(A))$
2. b^*g closed if $\text{cl}(A) \subseteq U$ whenever $A \subseteq U$ and U is b open
3. $(b^*g)^*$closed if $\text{cl}(A) \subseteq U$ whenever $A \subseteq U$ and U is b^*g open
4. θ closed if $A = \theta\, \text{cl}(A)$where $\theta\, \text{cl}(A) = \{ x \in X : \text{cl}(U) \cap A \neq \phi \forall\, U \in \tau \text{ and } x \in U\}$
5. δ closed if $A = \delta\, \text{cl}(A)$ where $\delta\, \text{cl}(A) = \{ x \in X, \text{int}(\text{cl}(U) \cap A \neq \phi, \forall\, U \in \tau \text{ and } x \in U\}$.

Definition 2.3: A function f: $(X, \tau) \rightarrow (Y, \sigma)$ is said to be almost contra continuous of $f^{-1}(V)$ is closed X, for every regular open set V in Y.

3. ALMOST CONTRA $G_{(b*g)*}$CONTINUOUS FUNCTION

Definition 3.1: A subset A of (X,τ,G) is called $G_{(b*g)*}$ closed if $\phi(A) \subseteq U$ whenever $A \subseteq U$ and U is b^*g open in X.

Definition 3.2: A function f: $(X,\tau,G) \rightarrow (Y, \sigma)$ is almost contra $G_{(b^*g)^*}$ continuous of $f^{-1}(V)$ is $G_{(b*g)*}$ closed in X, for every regular open set V of Y.

Example 3.3: Let $X = \{a, b, c,\}$, $\tau = \{ \phi, \{a\}, \{b\}, \{a, b, \}, \{a, c\}, X\}$ $G = \{ \{b, c\}, X\}$
Define f: $(X, \tau, G) \rightarrow (X, \tau)$ to be the identity function f is almost contra $G_{(b*g)*}$continuous.

Theorem 3.4: If $f: X \rightarrow Y$ is contra $G_{(b*g)*}$ continuous, then it is almost contra $G_{(b*g)*}$ continuous.

Proof: The proof is obvious, as every regular open set is open set.
The converse of the above theorem need not be true can be seen from the following example.

Example 3.5: Let $X = \{a, b, c,\}$, $\tau = \{ \phi, \{a, b\}, X\}$ $G = \{\{b\}, \{a, b\}, \{b, c\}, X\}$
Let f: $(X, \tau, G) \rightarrow (X, \tau)$ be the identity function f is almost contra $G_{(b*g)*}$continuous but not contra $G_{(b*g)*}$continuous as $f^{-1}(\{a, b\}) = \{a, b\}$ is not $G_{(b*g)*}$ closed.

Theorem 3.6:

1. Every almost contra continuous function is almost contra $G_{(b*g)*}$ continuous
2. Every almost contra $(b^*g)^*$ continuous function is almost contra $G_{(b*g)*}$continuous
3. Every almost contra θ continuous function is almost contra $G_{(b*g)*}$ continuous
4. Every almost contra δ continuous function is almost contra $G_{(b*g)*}$ continuous

Proof: Obvious
Converse of the above statements need not be true can be seen from the following examples.

Example 3.7: Refer example 3.3
f is almost contra $G_{(b*g)*}$ continuous but not almost contra continuous as $f^{-1}(\{a\}) = \{a\}$ is not closed in X.

Example 3.8: Refer example 3.3
f is almost contra $G_{(b*g)*}$ continuous but not almost contra (b*g)* continuous as $f^{-1}(\{a\}) = \{a\}$ is not (b*g)* closed.

Example 3.9 : Refer example 3.3
f is almost contra $G_{(b*g)*}$ continuous but not almost contra θ continuous as $f^{-1}(\{a\}) = \{a\}$ is not θ closed.

Example 3.10 : Refer example 3.3
f is almost contra $G_{(b*g)*}$ continuous but not almost contra δ continuous as $f^{-1}(\{a\}) = \{a\}$ is not δ closed.

Theorem 3.11: Let arbitrary union of $G_{(b*g)*}$ open sets be $G_{(b*g)*}$open in X. The following are equivalent for a function $f: (X, \tau, G) \to (Y, \sigma)$

1. f is almost contra $G_{(b*g)*}$ continuous
2. For every regular closed set F of Y, $f^{-1}(F)$ is $G_{(b*g)*}$ open in X
3. For each $x \in X$ and each regular closed set F of Y containing f(x), there exists $G_{(b*g)*}$ open set U containing x in X such that $f(U) \subseteq F$
4. For each $x \in X$ and each regular open set V of Y not containing f(x), there exists a $G_{(b*g)*}$ closed set K in X not containing x such that $f^{-1}(V) \subseteq K$.

Proof : 1) ⇔ 2) is obvious
2) ⇒ 3) Let F be a regular closed set in Y containing f(x). This implies $x \in f^{-1}(F)$. By 2) $f^{-1}(F)$ is $G_{(b*g)*}$ open in X containing x. Let $U = f^{-1}(F)$. This implies U is $G_{(b*g)*}$ open in X containing x and $f(U) = f(f^{-1}(F)) \subseteq F$.
3) ⇒ 2) Let F be regular closed in Y containing f(x). This implies $x \in f^{-1}(F)$.
From 3) there exists $G_{(b*g)*}$ open set U_x in X containing x such that $f(U_x) \subseteq F$. That is $U_x \subseteq f^{-1}(F)$. That's $f^{-1}(F) = U[U_x : x \in f^{-1}(F)]$. This is union of $G_{(b*g)*}$ open sets. So $f^{-1}(F)$ is $G_{(b*g)*}$ open in X.
3) ⇒ 4) Let V be regular open set in Y not containing f(x). Then Y – V is a regular closed set in Y containing f(x). From 3) then exists $G_{(b*g)*}$ open set U in X containing x such that $f(U) \subset Y - V$. This implies $U \subseteq f^{-1}(Y - V) = X - f^{-1}(V)$. Hence $f^{-1}(V) \subseteq X - U$. Let $K = X - U$. Then K is $G_{(b*g)*}$ closed not containing x such that $f^{-1}(V) \subseteq K$.
4) ⇒ 3) Let F be regular closed set in Y containing f(x). Then Y – F is a regular open set in Y not containing f(x). From 4), there is $G_{(b*g)*}$closed set K not containing x such that $f^{-1}(Y - F) \subseteq K$. That is $X - f^{-1}(F) \subseteq K$. Hence $X - K \subseteq f^{-1}(F)$. That is $f(X - K) \subseteq F$. Let $U = X - K$. U is $G_{(b*g)*}$ open containing x such that $f(U) \subseteq F$.

Theorem 3.12: The following are equivalent for a function $f: (X, \tau, G) \to (Y, \sigma)$

1. f is almost contra $G_{(b*g)*}$ continuous
2. $f^{-1}(int(cl(G)))$ is $G_{(b*g)*}$ closed in X for every open set G of Y
3. $f^{-1}(cl(int(F)))$ is $G_{(b*g)*}$ open in X for every open set F of Y

Proof: 1) ⇔ 2) Let G be open in Y. Then int(cl(G)) is regular open in Y.
By 1) $f^{-1}(int(cl(G)))$ is $G_{(b*g)*}$ closed in X.
2) ⟹ 1) Let V be regular open in Y. Then $f^{-1}(V) = f^{-1}(int(cl(V)))$ is $G_{(b*g)*}$closed in X, as V is open in Y. So, f is almost contra $G_{(b*g)*}$ continuous.
1) ⟹ 3) Let F be closed in Y. Then $cl(int(F))$ is regular closed in Y. By 1) $f^{-1}(cl(int(F)))$ is $G_{(b*g)*}$ open in X.
3) ⟹ 1) Obvious.

Definition 3.13: A function $f: (X, \tau, G) \to (Y, \sigma)$ is said to be R- map if $f^{-1}(V)$ is regular open for each regular open set V of Y.
Theorem 3.14: If $f: (X, \tau, G) \to (Y, \sigma)$ is almost contra $G_{(b*g)*}$ continuous and almost continuous, the f is an R-map.

Proof: Let $V \in RO(Y)$. Then $f^{-1}(V)$ is $G_{(b*g)*}$ closed and open. Then $f^{-1}(V)$is regular open in X. So, f is an R - Map.

Definition 3.15: A function $f: (X, \tau, G) \to (Y, \sigma)$ is said to be perfectly continuous if$f^{-1}(V)$is clopen for each open set V of Y.

Theorem 3.16: For two functions $f:(X,\tau,G)\rightarrow(Y,\sigma)$ and g : $Y\rightarrow Z$, let $gof: X\rightarrow Z$ be a composition function. Then the following hold.

1. If f is almost contra $G_{(b*g)*}$ continuous and g is an R-map, then g o f is almost contra $G_{(b*g)*}$continuous
2. If f is almost contra $G_{(b*g)*}$ continuous and g is perfectly continuous, the g o f is almost $G_{(b*g)*}$ continuous and almost contra $G_{(b*g)*}$ continuous
3. If f is contra $G_{(b*g)*}$ continuous and g is almost continuous, then g o f is almost contra $G_{(b*g)*}$continuous.

Proof :

1. Let V be regular open in Z. Then $g^{-1}(V)$ is regular open in Y. As f is almost contra $G_{(b*g)*}$ continuous. $(gof)^{-1}(V) = f^{-1}(g^{-1}(V))$ is $G_{(b*g)*}$ closed in X.
2. Let V be regular open in Z. Then $g^{-1}(V)$ is clopen in Y. That is $g^{-1}(V)$ is regular open and regular closed in Y. So, $(gof)^{-1}(V) = f^{-1}(g^{-1}(V))$ is $G_{(b*g)*}$ open and $G_{(b*g)*}$closed in X
3. Let V be regular open Z. $g^{-1}(V)$ is open in Y. As f is contra $G_{(b*g)*}$ continuous $(gof)^{-1}(V) = f^{-1}(g^{-1}(V))$ is $G_{(b*g)*}$ closed in X.

Definition 3.17: A grill topological space X is said to be $T_{G(b*g)*}$space if every $G_{(b*g)*}$ open set in X is open in X.

Definition 3.18: Let $f: X\rightarrow Y$ be contra $G_{(b*g)*}$ continuous and $g: Y\rightarrow Z$ be $G_{(b*g)*}$ continuous. If Y is a $T_{G(b*g)*}$ space, then g o $f:(X,\tau,G)\rightarrow(Z,ή)$ is almost contra $G_{(b*g)*}$continuous.

Proof: Let V be regular open in Z. Then $g^{-1}(V)$ is $G_{(b*g)*}$ open in Y. As Y is $T_{G(b*g)*}$space $g^{-1}(V)$ is open in Y. So, $(gof)^{-1}(V) = f^{-1}(g^{-1}(V))$ is $G_{(b*g)*}$ closed in X.

Definition 3.19: A function $f : X\rightarrow Y$ is said to be strongly $G_{(b*g)*}$ open (strongly $G_{(b*g)*}$closed) if f(U) is $G_{(b*g)*}$ open ($G_{(b*g)*}closed$) for every $G_{(b*g)*}$ open ($G_{(b*g)*}$ closed) set U of X.

Theorem 3.20: If$f : (X,\tau,G)\rightarrow(Y,\sigma)$is surjective and strongly $G_{(b*g)*}$ open (strongly $G_{(b*g)*}$closed) g : $Y\rightarrow Z$ is a function such that $gof : (X,\tau,G)\rightarrow(Y,\sigma)$is almost contra $G_{(b*g)*}$ continuous, then g is almost contra $G_{(b*g)*}$ continuous.

Proof: Let V be regular closed (regular open) set in Z. As g o f is almost contra $G_{(b*g)*}$ continuous $(gof)^{-1}(V) = f^{-1}(g^{-1}(V))$ is $G_{(b*g)*}$ open ($G_{(b*g)*}$ closed). Since f is surjective and strongly $G_{(b*g)*}$ open (strongly $G_{(b*g)*}$closed) $f(f^{-1}(g^{-1}(V))) = g^{-1}(V)$ is $G_{(b*g)*}$open ($G_{(b*g)*}$ closed). Hence g is almost contra $G_{(b*g)*}$continuous.

Definition 3.21: A function $f:(X,\tau,G)\rightarrow(Y,\sigma)$ is said to be weakly$G_{(b*g)*}$continuous, if for each $x\in X$ and each open set V of Y, containing f(x) there exists a $G_{(b*g)*}$open set U of X containing x such that $f(U)\subseteq cl(V)$.

Theorem 3.22: If a function $f:(X,\tau,G)\rightarrow(Y,\sigma)$ is almost contra $G_{(b*g)*}$ continuous, then f is weakly $G_{(b*g)*}$ continuous function.

Proof: Let $x\in X$ and V be an open set containing f(x). Then cl(V) is regular closed in Y containing f(x).. As f is almost contra $G_{(b*g)*}$ continuous, $f^{-1}(cl(V))$ is $G_{(b*g)*}$ open in X containing x. Let $U = f^{-1}(cl(V))$. Then $f(U)\subseteq f(f^{-1}(cl(V)))\subset cl(V)$. Hence f is almost weakly $G_{(b*g)*}$ continuous.

Definition 3.23: A grill topological space X is called locally $G_{(b*g)*}$ indiscrete, if every $G_{(b*g)*}$ open set is closed in X.

Theorem 3.24: If a function $f:(X,\tau,G)\rightarrow(Y,\sigma)$ is almost contra $G_{(b*g)*}$ continuous and X is locally $G_{(b*g)*}$ indiscrete, then f is almost continuous.

Proof: Let V be regular closed in Y. So $f^{-1}(V)$ is $G_{(b*g)*}$ open in X. As X is locally $G_{(b*g)*}$ indiscrete. $f^{-1}(V)$ is closed in X. Hence f is almost continuous.

4.$G_{(b*g)*}$*Regular* graphs

Definition 4.1: For a function $f:(X,\tau,G)\rightarrow(Y,\sigma)$,the subset$\{(x,f(x):x\in X\}\subseteq XxY$ is called the graph of f and is denoted by G(f).

Definition 4.2: A graph G(f) of a function $f:(X,\tau,G)\rightarrow(Y,\sigma)$, is said to be $G_{(b*g)*}$ regular if for each $\{(x,y)\in(XxY)-G(f)$, there exists $G_{(b*g)*}$ closed set U in X containing x and $V\in RO(Y)$ containing y such that $(UxV)\cap G(f) = \phi$.

Lemma 4.3: The following properties are equivalent for a graph G(f) of a function:
1. G(f) is $G_{(b*g)*}$ regular
2. For each point $(x,y) \in (XxY) - G(f)$, there exist a $G_{(b*g)*}$ closed set U in X containing x and $V \in RO(Y)$ containing y such that $f(U) \cap V = \emptyset$.

Proof:
1) $\Longrightarrow$ 2) Let $(x,y) \in (XxY) - G(f)$. Then there exists a $G_{(b*g)*}$ closed set U in X containing x and $V \in RO(Y)$ containing y such that $(UxV) \cap G(f) = \phi$. That is $V \cap f(X) = \phi$. That is $V \cap f(U) = \phi$.
2) $\Longrightarrow$ 1) Assume 2). $y \in V, y \in Y - f(X)$. That is $y \neq f(x)$ for any $x \in X$. That is $V \cap f(X) = \phi$. This implies $(UxV) \cap (Xxf(X)) = \phi$. That is $(UxV) \cap G(f) = \phi$.

Theorem 4.4: If $f : X \to Y$, is almost contra $G_{(b*g)*}$ continuous and Y is T_2, then G(f) is $G_{(b*g)*}$ regular in X x Y.

Proof:
Let Y be T_2. Let $(x,y) \epsilon (XxY) - G(f)$. It follows $f(x) \neq y$. As Y is T_2 there exist open sets V and W containing f(x) and y respectively such that $V \cap W = \phi$. Then $int(cl(V)) \cap int(cl(W)) = \phi$. Since f is almost contra $G_{(b*g)*}$continuous $f^{-1}(int(cl(V)))$ is $G_{(b*g)*}$ closed in X as int(cl(V)) is regular open in Y.
Let $U = f^{-1}\left(int(cl(V))\right)$. Then $f(U) \subseteq int(cl(V))$. So, $f(U) \cap int(cl(W)) = \phi$. Hence G(f) is $G_{(b*g)*}$regular in X x Y.
The intersection of two $G_{(b*g)*}$open sets need not be $G_{(b*g)*}$open. But in the following theorem, we assume that intersection of two $G_{(b*g)*}$ open sets is $G_{(b*g)*}$ open.

Theorem 4.5: Let $f : (X,\tau,G) \to (Y,\sigma)$ be a function and $g: (X,\tau) \to (XxY, \tau x\sigma)$, the graph function defined $g(x) = (x, f(x))$, for every $x \epsilon X$. Then f is almost $G_{(b*g)*}$ continuous if and only if g is almost $G_{(b*g)*}$continuous.

Proof:
Let g be almost $G_{(b*g)*}$ continuous. Let $x \epsilon X$ and $V \epsilon RO(Y)$ containing f(x). Then $g(x) = (x, f(x) \epsilon RO(XxY)$. As g is almost $G_{(b*g)*}$ continuous, there exist $G_{(b*g)*}$ open set U of X containing x such that $g(U) \subset XxY$. So, $f(U) \subseteq V$. Hence f is almost $G_{(b*g)*}$ continuous.
Conversely, let f be almost $G_{(b*g)*}$continuous. Let $x \in X$ and W be a regular open set of X x Y containing g(x). There exists $U_1 \in RO(X,\tau)$ and $V \in RO(Y,\sigma)$ such that $(x, f(x)) \in (U_1 xV) \subset W$. As f is almost $G_{(b*g)*}$ continuous there exists $U_2 \in RO(X,\tau)$ such thatx $\in U_2$ and $f(U_2) \subset V$. Let $U = U_1 \cap U_2$. We have $x \in U \in G_{(b*g)*}O(X,\tau)$ and $g(U) \subset (U_1 xV) \subset W$. This implies g is almost $G_{(b*g)*}$continuous.

5 Connectedness

Definition 5.1: A grill topological space X is calledG$_{(b*g)*}$ connected if X cannot be written as a disjoint union of two non-empty $G_{(b*g)*}$ open sets.

Definition 5.2: If $f : (X,\tau,G) \to (Y,\sigma)$ is an almost contra $G_{(b*g)*}$ continuous surjection and X is $G_{(b*g)*}$ connected then Y is connected.

Proof: Let Y be not connected. Then $Y = U_0 \cup V_0$ such that U_0 and V_0 are disjoint nonempty open sets. Let $U = int(cl(U_0))$ and $V = int(cl(V_0))$. Then U and V are disjoint nonempty regular open sets such that $Y = U \cup V$. As f is almost contra $G_{(b*g)*}$ continuous. $f^{-1}(U)$ and $f^{-1}(V)$ are $G_{(b*g)*}$closed sets of X. We have $X = f^{-1}(U) \cup f^{-1}(V)$such that $f^{-1}(U)$ and $f^{-1}(V)$ are disjoint. Since f is surjective. $f^{-1}(U)$ and $f^{-1}(V)$are nonempty. This implies X is not $G_{(b*g)*}$ connected. Hence Y is connected.

Theorem 5.3 : The almost contra $G_{(b*g)*}$ continuous image of $G_{(b*g)*}$ connected space is connected.

Proof : Let $f : (X,\tau,G) \to (Y,\sigma)$ be an almost contra $G_{(b*g)*}$ continuous function of a $G_{(b*g)*}$ connected space X onto a topological space Y. Suppose Y is not a connected space. The $Y = V_1 \cup V_2$ where V_1 andV_2 are disjoint nonempty open sets of Y. So, V_1 andV_2 are clopen in Y. As f is almost contra $G_{(b*g)*}$ continuous. $f^{-1}(V_1)$ and $f^{-1}(V_2)$ are $G_{(b*g)*}$ open in X. Also $f^{-1}(V_1)$ and $f^{-1}(V_2)$ are disjoint nonempty and $X = f^{-1}(V_1) \cup f^{-1}(V_2)$. This contradiction shows Y is connected.

Definition 5.4 : A grill topological space X is said to be $G_{(b*g)*}$ ultra connected if every two non empty $G_{(b*g)*}$closed subsets of X intersect.

Definition 5.5 : A topological space X is said to be hyper connected if every open set is dense.

Theorem 5.6 : If X is $G_{(b*g)*}$ ultra connected and $f:(X,\tau,G)\to(Y,\sigma)$ is almost contra $G_{(b*g)*}$ continuous surjection, then Y is hyperconnected.

Proof : Let Y be not hyper connected. So, there exists an open set V in Y such that V is not dense in Y. So, there exist nonempty regular open set: $B_1 = int(cl(V))$ and $B_2 = Y - cl(V)$ in Y. As f is almost contra $G_{(b*g)*}$ continuous $f^{-1}(B_1)$ and $f^{-1}(B_2)$ are disjoint $G_{(b*g)*}$ closed. This contradicts the $G_{(b*g)*}$ ultraconnectedness of X. Hence Y is hyperconnected.

6 Separation axioms

Definition 6.1 : A grill topological space X is said to be $G_{(b*g)*}T_1$ space if for any pair of distinct points x and y, there exist $G_{(b*g)*}$ open sets G and H such that $x\in G, y\notin G$ and $x\notin H, y\in H$.

Definition 6.2 : A space X is said to be weakly Hausdorff if each element of X is an intersection of regular closed sets.

Theorem 6.3 : If $f: X\to Y$ is almost contra $G_{(b*g)*}$ continuous injection and Y is weakly Hausdorff, then X is $G_{(b*g)*}T_1$.

Proof : Let Y be weakly Hausdorff. For any distinct points x and y in X, there exist V and W regular closed sets in Y such that $f(x)\in V, f(y)\notin V$, and $f(y)\in W$ and $f(x)\notin W$. Since f is almost contra $G_{(b*g)*}$ continuous, $f^{-1}(V)$ and $f^{-1}(W)$ are $G_{(b*g)*}$ open sets of X such that $x\in f^{-1}(V), y\notin f^{-1}(V)$ and $y\in f^{-1}(W), x\notin f^{-1}(W)$. This completes the proof.

Corollary 6.4 : If $f:(X,\tau,G)\to(Y,\sigma)$ is contra $G_{(b*g)*}$ continuous injection and y is weakly Hausdorff, then X is $G_{(b*g)*}T_1$.

Definition 6.5 : A topological space X is called Ultra Hausdorff space, if for every pair of distinct points x and y in X, there exist disjoint clopen sets U and V in X, containing x and y respectively.

Definition 6.6 : A grill topological space is said to be $G_{(b*g)*}T_2$ space if for any pair of distinct points x and y in X, there exist disjoint $G_{(b*g)*}$ open sets G and H such that $x\in G$ and $y\in H$.

Theorem 6.7 : If $f:(X,\tau,G)\to(Y,\sigma)$ is an almost contra $G_{(b*g)*}$ continuous injective function from space X into a Ultra Hausdorff space Y, then X is $G_{(b*g)*}T_2$.

Proof : Let x and y be distinct points in X. As f is injective $f(x)\neq f(y)$. As Y is Ultra Hausforff space, there exist disjoint clopen sets U and V of Y containing f(x) and f(y) respectively. Then $x\in f^{-1}(U)$ and $y\in f^{-1}(V)$ where $f^{-1}(U)$ and $f^{-1}(V)$ are disjoint $G_{(b*g)*}$ open sets in X. Hence the assertion.

Definition 6.8 : A topological space is called Ultra normal space, if each pair of disjoint closed sets can be separated by disjoint clopen sets.

Definition 6.9 : A grill topological space X is said to be $G_{(b*g)*}$ normal if each pair of disjoint closed sets can be separated by disjoint $G_{(b*g)*}$ open sets.

Theorem 6.10 : If $f:(X,\tau,G)\to(Y,\sigma)$ is an almost contra $G_{(b*g)*}$ continuous closed injection and Y is Ultra normal, then X is $G_{(b*g)*}$ normal.

Proof : Let E and F be disjoint closed subsets of X. As f is closed and injective f(E) and f(F) are disjoint closed sets in Y. Since y is Ultra normal, there exist disjoint clopen sets U and V in Y such that $f(E)\subset U$ and $f(F)\subset V$. This implies $E\subseteq f^{-1}(U)$ and $F\subseteq f^{-1}(V)$. As f is almost contra $G_{(b*g)*}$ continuous, $f^{-1}(U)$ and $f^{-1}(V)$ are disjoint $G_{(b*g)*}$ open sets in X. This completes the proof.

Lemma 6.11 : If $f:(X,\tau,G)\to(Y,\sigma)$ is an almost $G_{(b*g)*}$ continuous implies for each $x\in X$ and for every regular open set V of Y containing f(x), there exists $G_{(b*g)*}$ open set U in X containing x such that $f(U)\subseteq V$.

Proof : Let $f: X\to Y$ be almost $G_{(b*g)*}$ continuous. Let V be regular open in Y containing f(x). $f^{-1}(V)$ is $G_{(b*g)*}$ open in X containing x. Let $U = f^{-1}(V)$. This implies U is $G_{(b*g)*}$ open in X containing x and $f(U) = f(f^{-1}(V))\subseteq V$.

Theorem 6.12 : If $f:(X,\tau,G)\to(Y,\sigma)$ is an almost $G_{(b*g)*}$ continuous and Y is semiregular, then f is $G_{(b*g)*}$ continuous.

Proof : Let $x \in X$ and V be an open set of Y containing f(x). By the definition of semi regularity of Y, there exists a regular open set G of Y such that $f(x) \in G \subseteq V$. Since f is almost $G_{(b*g)*}$ continuous there exists $U \in G_{(b*g)*}O(X, x)$ such that $f(U) \subseteq G$. Hence we have $f(U) \subseteq G \subseteq V$. This shows f is $G_{(b*g)*}$ continuous.

7. Compactness

Definition 7.1 : A space X is said to be

1. $G_{(b*g)*}$ compact if every $G_{(b*g)*}$ open cover of X has finite sub cover
2. $G_{(b*g)*}$ clased compact if every $G_{(b*g)*}$ closed cover of X has a finite sub cover
3. Nearly compact if every regular open cover of X has a finite sub cover
4. Countably $G_{(b*g)*}$ compact if every countable cover of X by $G_{(b*g)*}$ open sets has a finite sub cover
5. Countably $G_{(b*g)*}$ closed compact if every countable cover of X by $G_{(b*g)*}$ closed sets has a finite subcover
6. Nearly countably compact if every countable cover of X by regular open sets has a finite sub cover
7. $G_{(b*g)*}$ Lindelof if every $G_{(b*g)*}$ open cover of X has a countable sub cover
8. $G_{(b*g)*}$ closed Lindelof if every $G_{(b*g)*}$ closed cover of X has a countable sub cover
9. Nearly Lindlof if every regular open cover of X has countable sub cover
10. S – Lindlof if every cover of X by regular closed sets has a countable sub cover
11. Countably S closed if every countable cover of X by regular closed sets has a finite sub cover
12. S – closed if every regular closed cover of X has a finite sub cover

Theorem 7.2 : Let $f : (X, \tau, G) \rightarrow (Y, \sigma)$ be an almost contra $G_{(b*g)*}$ continuous surjection. Then the following properties hold:

1. If X is $G_{(b*g)*}$ closed compact, the Y is nearly compact
2. If X is countably $G_{(b*g)*}$ closed compact, then Y is nearly countably compact
3. If X is $G_{(b*g)*}$closed Lindelof, then Y is nearly Lindelof

Proof : 1) Let $\{ V_\alpha : \alpha \epsilon I\}$ be any regular open cover of Y. As f is almost contra $G_{(b*g)*}$ continuous. $\{f^{-1}(V_\alpha) : \alpha \epsilon I\}$ is $G_{(b*g)*}$ closed cover of X. Since X is $G_{(b*g)*}$ closed compact, there exists a finite subset I_0 ofIsuch that $X = \cup\{f^{-1}(V_\alpha): \alpha \epsilon I_0\}$. As f is surjective,$Y = \cup\{V_\alpha : \alpha \epsilon I_0\}$, which is a finite sub cover of Y. Hence Y is nearly compact.
The proof of 2) and 3) are similar.

Theorem 7.3 : Let $f : (X, \tau, G) \rightarrow (Y, \sigma)$ be an almost contra $G_{(b*g)*}$ continuous surjection. Then the following hold:

1. If X is $G_{(b*g)*}$ compact then Y is S-closed
2. If X is countably $G_{(b*g)*}$ compact, then Y is countably S-closed
3. If X is $G_{(b*g)*}$ Lindelof, then Y is S- Lindelof

Proof : 1) Let $\{V_\alpha : \alpha \epsilon I\}$ be any regular closed cover of Y. As f is almost contra $G_{(b*g)*}$ continuous $\{f^{-1}(V_\alpha) : \alpha \epsilon I\}$ is $G_{(b*g)*}$ open cover of X. Since X is $G_{(b*g)*}$ compact, there exist a finite subset I_0 of I such that $X = \cup\{f^{-1}(V_\alpha) : \alpha \epsilon I_0\}$. As f is surjective $Y = \cup\{V_\alpha: \alpha \epsilon I_0 \}$ is a finite sub cover for Y. This shows Y is S- closed.
The proof of 2) and 3) are similar.

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